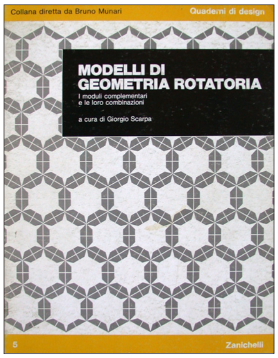
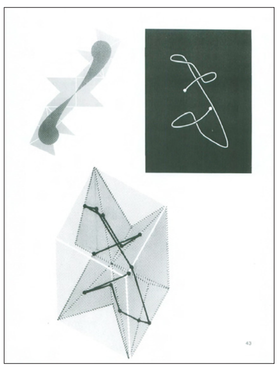
ROTATIONAL GEOMETRY AS A TEACHING TOOL : APPLYING THE WORK OF GIORGIO SCARPA

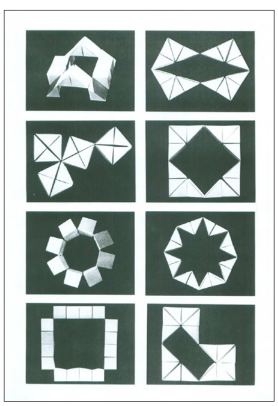
PINO TROGU – SAN FRANCISCO STATE UNIVERSITY

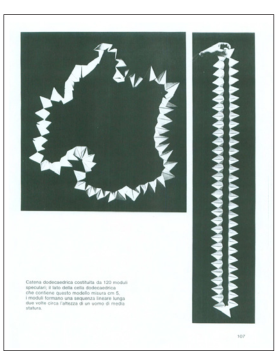
SIXTH INTERNATIONAL CONFERENCE ON DESIGN PRINCIPLES AND PRACTICES

LOS ANGELES, JANUARY 21, 2012









Giorgio Scarpa, Modelli di Geometria Rotatoria [Models of Rotational Geometry]. Bologna: Zanichelli, 1978. Cover and sample pages: 43, 77, 107.

Genesis of form.
Motion is at the root of all growth
Paul Klee

Although rotational geometry is a difficult field of mathematics available only to specialists, the physical models that apply its principles are highly useful for courses in sketching and drawing. Students at San Francisco State have found rotational geometry to be one of the most valuable segments of the drawing course, offering such remarks as: "I feel that this project used all of the skills that we learned in class, from drawing the basic shape in orthographic/ axonometric views to the cubic modules in the perspective." "This project challenged my design thinking by taking a 2D object and rendering it in a 3D environment."
I was able to teach this segment of the course thanks to the teaching and writings of the Italian scholar Giorgio Scarpa (b 1938). This presentation introduces his work to English-speaking specialists, and illustrates how the subject can be made useful to design students.
Giorgio Scarpa taught Descriptive Geometry at the Istituto d'Arte of Oristano and Faenza, Italy, and Theory of Perception at the Istituto Superiore Industrie Artistiche (ISIA) in Faenza. His book *Modelli di Geometria Rotatoria*, which was part of a design series edited by the late Italian designer Bruno Munari, is the basis of this study. This teaching unit in drawing for design uses and applies Scarpa's principles and methods, and tests their validity through the construction of physical models built by the students. Through this process, students learn to apply a visual grammar based on rotational movements and folding which transform two-dimensional shapes into three-dimensional solids. These solids are modules derived from the sectioning of regular polyhedra such as the cube. In theory, any regular polyhedron can be used as the basis for the section.
In this study only the cube is used, due to its simple, intuitive symmetry.
Drafting and Sketching for Design is a required course for all students entering the Design and Industry Department at San Francisco State University. In the class, all drawing is done by hand with drafting tools and free hand sketching. The class covers orthographic projections, axonometric projections, and perspective. These techniques are also explored within a unit called Cube Section.
The unit begins with the simple problem: dissect a 4″ x 4″ x 4″ cube into two or three solid modules (polyhedra), having identical surface area, volume, and shape. The three-dimensional modules that will form the final cube can be connected at a later time by means of hinges. The connected modules can be arranged into open or closed chains. The modules may or may not fold back into a minimum volume enclosure depending on the type and orientation of the hinges used. The materials used in this process are pencil, paper and tape or glue.
While the students are able to improve their manual skills through the use of these materials, the alternative use of CAD and 3D printing would allow for faster testing of the various configurations.
We'll call the process for the section that divides the cube into two modules the "twin" section. The process that divides the cube into three modules will be called the "triplet" section.
Text and images in this handout are adapted from an article by the same title. More details can be found at the URLs below. Thank you.

Pino Trogu
San Francisco State University
trogu@sfsu.edu
design.sfsu.edu
trogu.com

Cube section – `trogu.com/projects/getProject/20051225082001/project_html`
Giorgio Scarpa – `res.trogu.com/scarpa`
This handout and slides – `trogu.com/Documents/conference/design-principles-and-practices`

TWIN SECTION

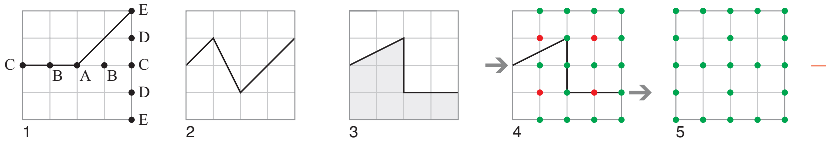


Fig. 1 — Start the section of the square at the mid-point on the left edge. End the section at any point on the grid on the right edge. Segments can go through red dots but not end on them.

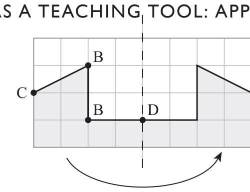


Fig. 2 — *Mirror the original shape (square) along the right edge of the square. The right edge is the axis of symmetry. The points on the section are labeled and the distances from those points to the center of the cube can be found in the cross section drawings. See Appendix A.*

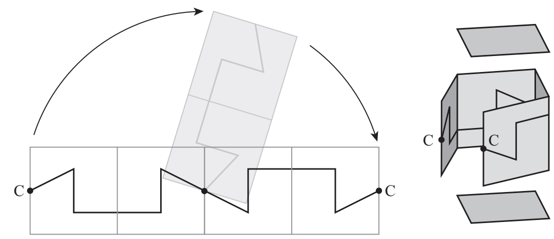


Fig. 3 — Rotate the resulting two squares by 180 degrees with center on the midpoint of the right edge. The resulting 4-square group represents the external surface of the cube, minus the top "lid" and the bottom "base". When folded into 3D space, the beginning left point and the ending right point of the section will match (point C).

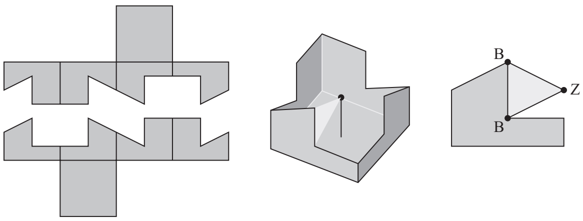


Fig. 4 — Each foldout shape is exactly half the external surface of the cube. Each segment along the section will be connected to the center of the cube to determine the triangles needed for the foldout surface of the internal section. Z: center of cube. BB: base of one of the triangles (BBZ).

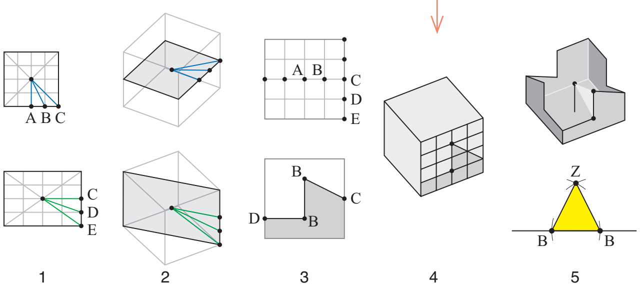


Fig. 5 — Points on the face of the cube that are needed for the construction of the triangles are labeled A through E. In the example, only points B, C, and D are used (5.3). The center of the cube is labeled Z. The two points (ends) of each segment on the face are connected to the center point, forming unique triangles. In the example, two B points are connected to the center of the cube (5.5). By constructing a series of adjacent triangles, we determine the internal surface of the sectioned cube. The resulting flat co-planar foldout shape will then be folded into the appropriate configuration, matching the center and the segments along the faces.

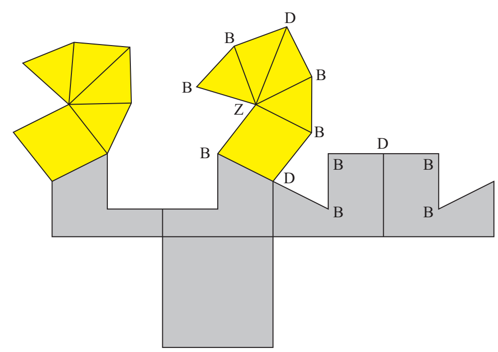


Fig. 6 — Yellow foldout with vertexes marked. These vertexes (yellow) will be matched to points on the external surface (gray).

The students document the modules in a series of drawings. The drawings are done by hand. The examples shown below were drawn on the computer for ease of reproduction.

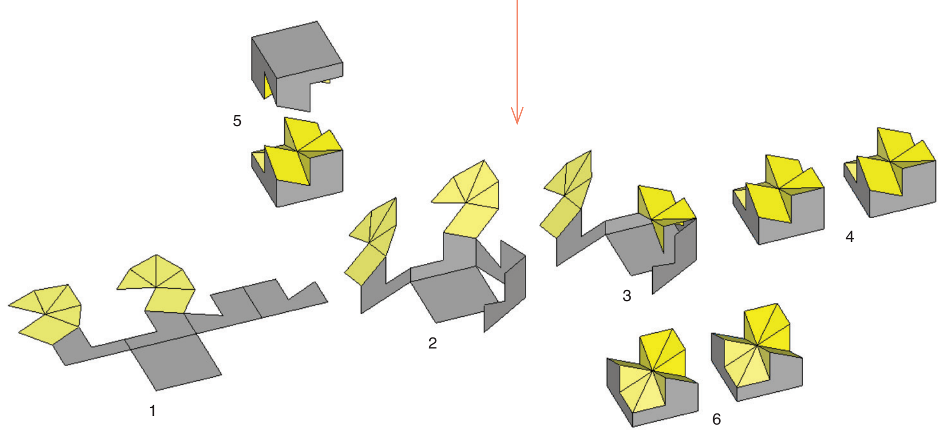


Fig. 7 — Folding sequence of the module from flat polygon to completed solid.

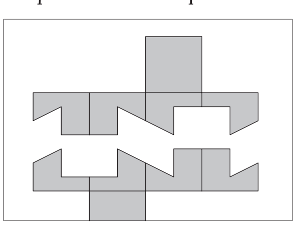


Fig. 8 — External surface fold-out.

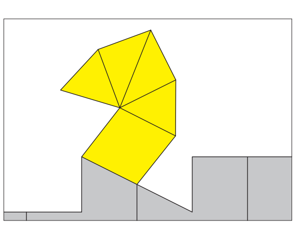


Fig. 9 — Internal surface fold-out.

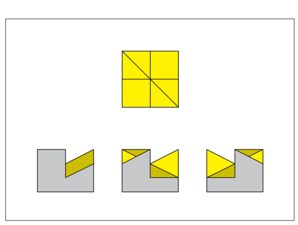


Fig. 10 — Orthographic views.

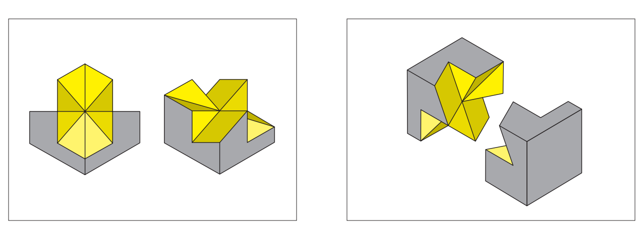


Fig. 11*A* and **11B** — Isometric views of the modules.

TRIPLET SECTION

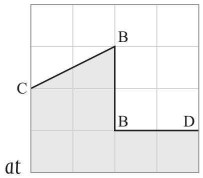


Fig. 12 — Given a square 4″ x 4″ and its modular grid, draw a segmented line that divides the square (face of the cube) into two separate parts.

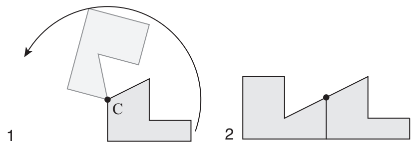


Fig. 13 — Rotate one of the two resulting shapes 180° with center in C so that the two shapes now share a boundary that is part of the edge of the square. In addition to rotation, other operations such as translations (movements in a straight line along an axis) are possible in order to achieve the proper section.

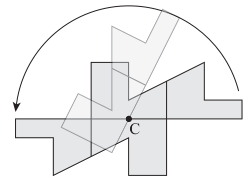


Fig. 14 — Rotate the new shape 180° with center on a new point C. We now have a total surface area of two squares.

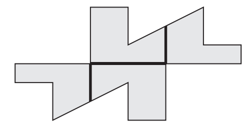


Fig. 15 — This new continuous four-part shape occupies exactly one third of the external surface of the cube. Common boundaries (thick lines in the illustration) will be folded 90° in 3D space.

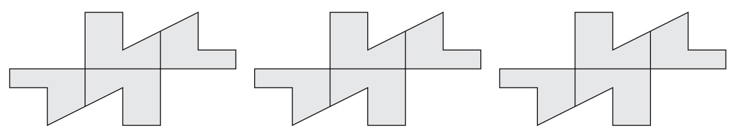


Fig. 16 — Repeat the process to obtain two additional shapes for a total of three identical shapes. We now have, on the plane, the three shapes (four parts in each) comprising the full external surface of the cube.

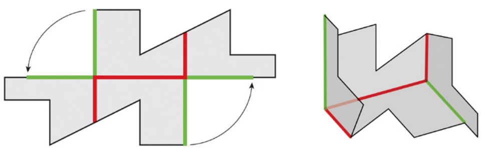


Fig. 17 — With each shape, fold and connect along the shared boundaries and edges of the squares. The 90-degree folding will yield the three pieces (each 1/3 of the cubic space) of the puzzle. These can be fitted back together to form the cube.

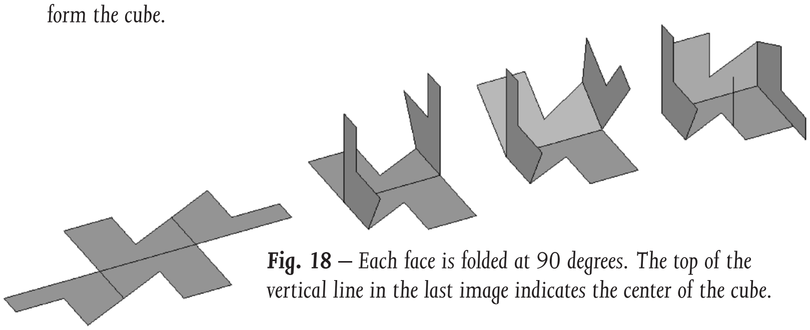


Fig. 18 — Each face is folded at 90 degrees. The top of the vertical line in the last image indicates the center of the cube.

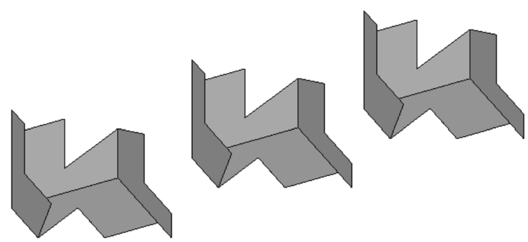


Fig. 19 — Take the three identical modules forming the cube and set them side by side. They must be identical in shape and orientation. The next step is to preserve the equality of the modules inside the cube (three equal volumes) as it is outside (three equal external surfaces). The internal planes of the modules (triangles) need to be determined, in order to fill the space completely.

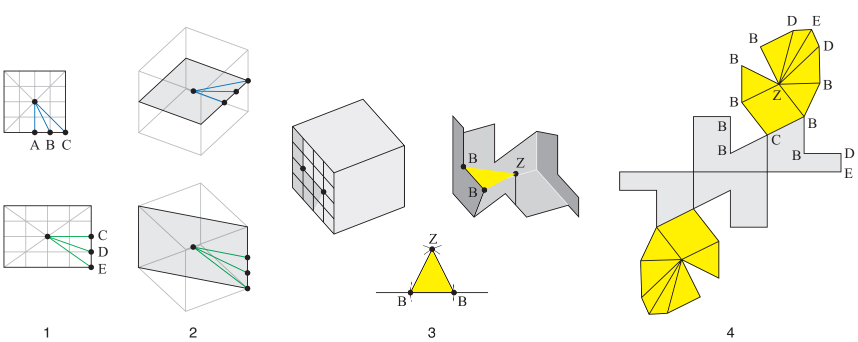


Fig. 20 — For each segment that is part of the original section, a triangle will be constructed where the base is the segment itself, and the two sides are lines connecting the segment with the center of the cube (20.3). This is similar to Fig. 5 and 6.

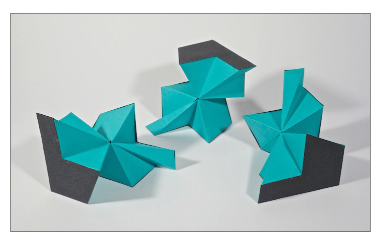


Fig. 23 — Actual modules built with card stock. A combination of blue and yellow board is used later in the six-module version of this cube.

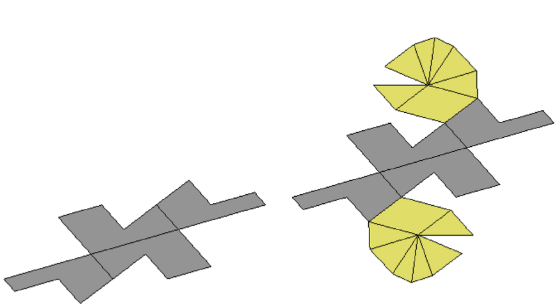


Fig. 21 — The external surface (gray) and the internal surface (yellow), flat, at the halfway point of folding, and completely folded.

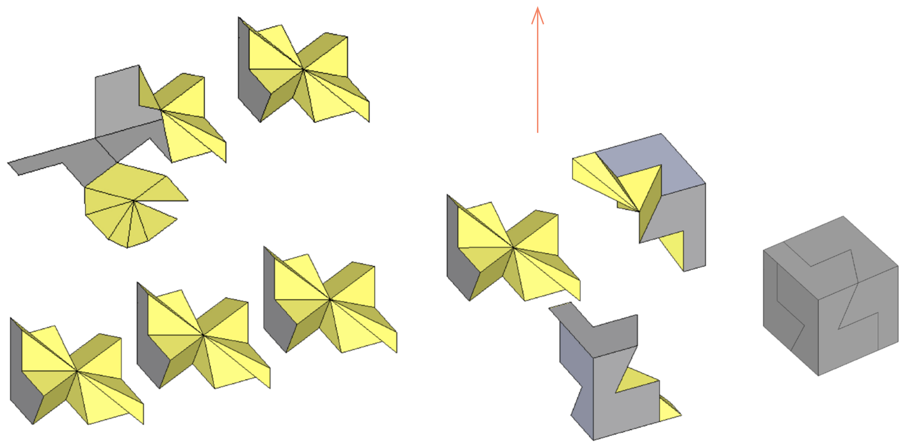


Fig. 22 — The three modules shown side by side and in exploded view. All modules are identical and right-handed. Completed cube at right.

HINGES AND ROTATIONS

Before the three modules are hinged together, they are sectioned into a total of six identical modules. This further division is helpful when the three finished modules do not fit back together physically, usually because of collisions due to undercuts in the design of the section.

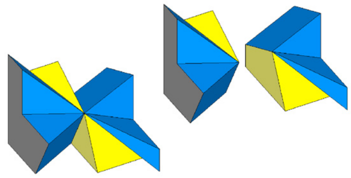


Fig. 24 – The 1/3 module split in half. The resulting 1/6 modules are identical and right-handed.

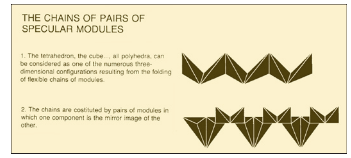


Fig. 25 – *In a typical chain constructed by Scarpa and illustrated in his book, "pairs" of symmetrical modules form the basic structure of the chain. The modules are repeated and hinged together along symmetry axes (Modelli, page 62).*

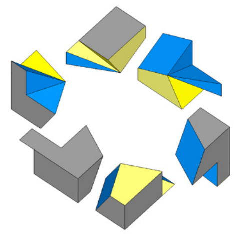


Fig. 27 – Exploded view of the six modules.

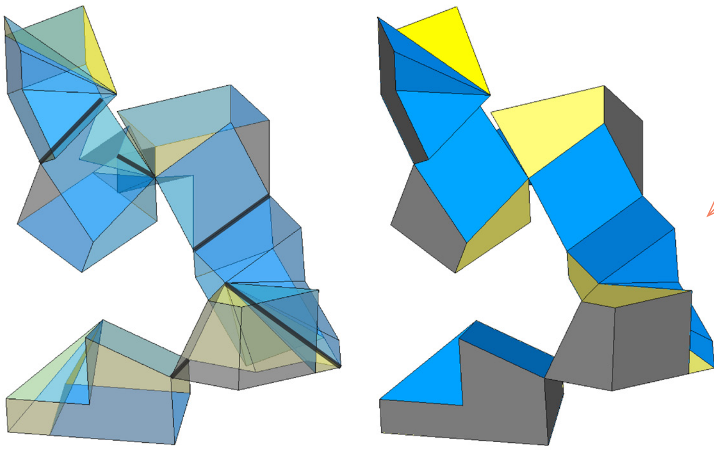


Fig. 28 – The six modules have been hinged along the cube's half diagonals and other segments on the faces. The hinges are shown as dark thick lines in the view on the left. Four such groups are connected together in the "closed-chain" configuration used for the model seen in Fig. 29 and Fig. 30.

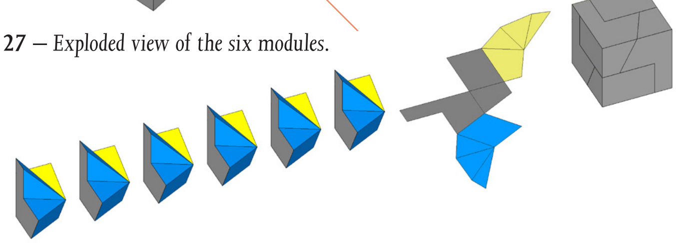


Fig. 26 – The new six modules. Internal (yellow and blue) and external (gray) fold-out surface. Completed cube on the right.

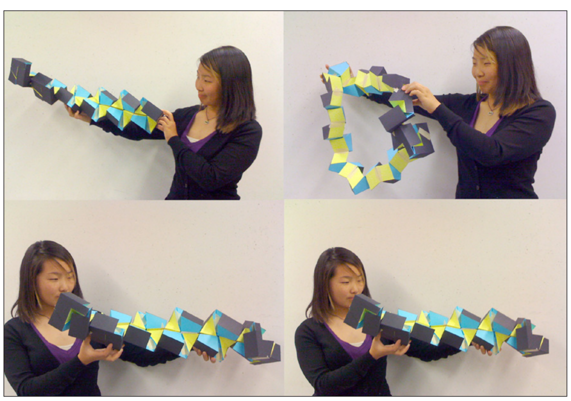


Fig. 29 – *In this configuration the selected placement of the hinges produced a closed chain where the modules cannot fold back into their minimal volume of 2 x 2 x 1 cubes. Chain is composed of 24 identical modules. Each module occupies 1/6 of one cube. Four complete cubes compose the chain. Model by Florence Gold Yuen, SFSU.*

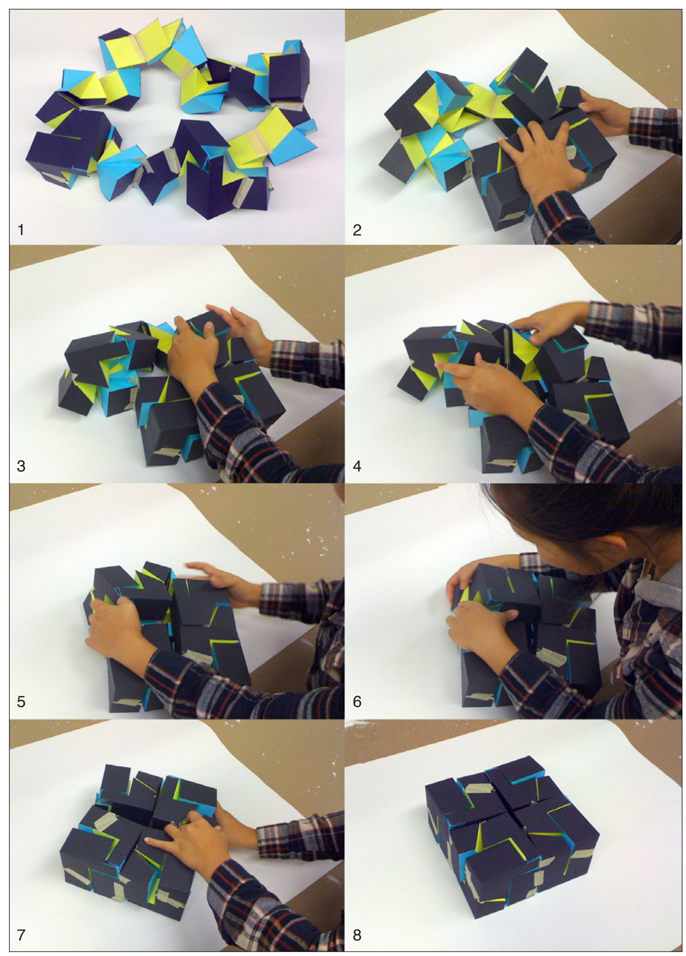
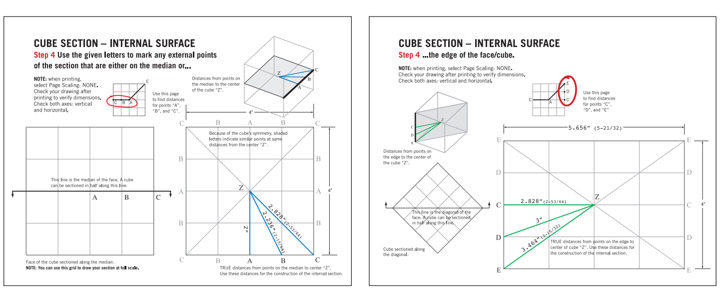


Fig. 30 – The same 24 modules seen in Fig. 29 were used to create this chain, which folds back into the minimum volume of 2 x 2 x 1 cubes. The location and spatial orientation of the hinges needs to be formalized and mapped. *We can expand this configuration into a larger volume composed of eight cubes, having 48 modules hinged together in a similar sequence. Model by Florence Gold Yuen, SFSU.*



Appendix A – *Diagrams showing geometric construction used to determine the internal measurements of the modules.*