Collana diretta da Bruno Munari

Quaderni di design

MODELLI DI GEOMETRIA ROTATORIA

I moduli complementari e le loro combinazioni

a cura di Giorgio Scarpa

5

Zanichelli

Giorgio Scarpa

Models of rotatory geometry

SCARPA
Modelli di
geometria
rotatoria
ZANICHELLI

2097000

Editor: Vanna Rossi

Graphic assistance: Giovanna Fiorillo

First edition: october 1978

Reprints:
1 2 3 4 1978 1979 1980
1981

Printed in Bologna
by Graficoop, Via dei Vestiari, 14
for N. Zanichelli Editore S.p.A.
Via Irnerio, 34, Bologna

Design notebooks 5

Giorgio Scarpa

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Translated by Pino Trogu

Zanichelli

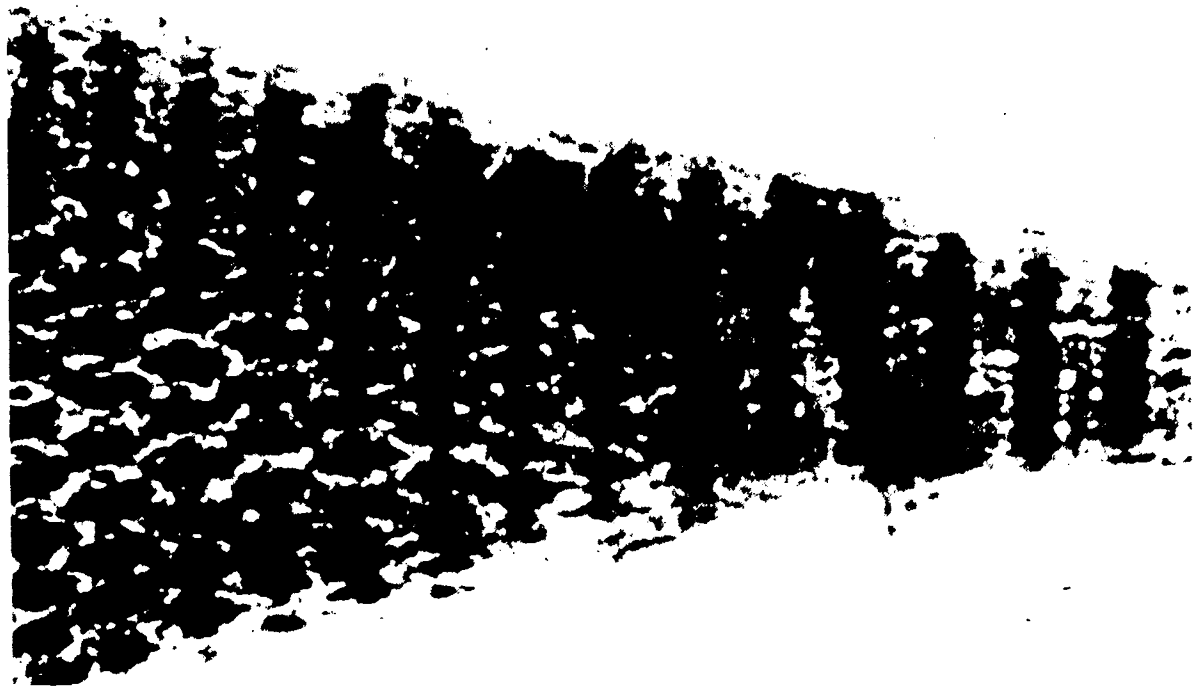
SUMMARY

The movement of rotation in nature

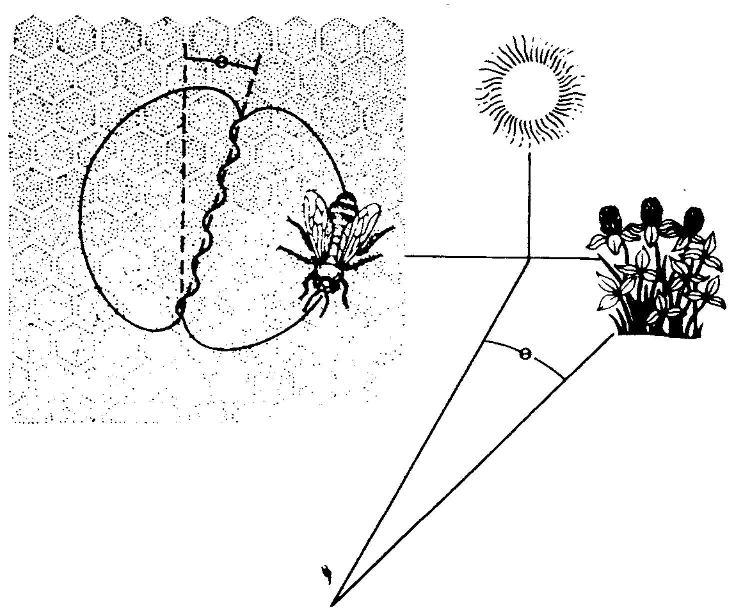
The movement of rotation is found throughout the world. The earth and the other planets rotate around themselves and around the sun. In its voyage through space, the solar system rotates around the center of our Galaxy and all the other galaxies rotate around their centers. Even inside atoms there is something similar to the rotation movements, of the electrons around themselves and around the nucleous, of the protons and neutrons in the nucleous.
The movements of rotation describe circles or ellipses or other closed trajectories. If we overlap a process of growth or an expansion on the rotation, the circles become spirals. Many spiral forms can be found in nature, from Galaxies to flowers to shells, even inside the cells.
The sheates of fats and proteins that cover certain nervous fibers grow rotating around the fiber and hence assume a spiral form. Even certain chemical reactions progress rotating and manifest themselves with colored waves, in the form of circles and spirals
If we overlap a translation in space to the rotation movement in a plane, the resulting figure is a cylindric helix. Helicoidal forms are very common in the living world.
The deoxyribonucleic acid molecula, a fundamental substance for the life on earth, has a structure formed by two entwined helixes, wrapped around a common axis. The helix is also one of the principal structures of the proteins, another class of substances indispensable for life: a helix-shaped protein, the keratin is the prime constituent of wool, hair; the myosin, a protein of the muscles, is formed by pairs of entwined helixes; a triple helix is the basic structure of collagen, which forms tendons, the strongest biological fibers. In certain viruses, the proteins arrange themselves in a helix manner, constructing an empty cylinder, within which we find the helix of the nucleic acid. In the cells of the *Spirogyra* algae we notice a long green stripe going around in a helix: these are the cloroplasts, where the photosyntetic activity takes place. Many organisms move following helicoidal trajectories. *Euglena gracilis,* a micro-organism that moves pushed up by a flagellum, moves with a screwing movement around itself, describing a helix on a cone; the flagellum itself is made of helicoidal proteins and moves almost like a helix.
Digging animals, like moles or certain earth worms, proceed in the digging with rotatory motions, like a drill or a bottle opener.
Certain birds that nest in bamboo swamp areas, make a series of rotatory movements with their whole bodies while building the nest, after landing on the center of a bunch of grass fragments and stems, on a biforked branch. Sometimes these birds start their rotatory movements even before having picked a single grass stem, to prove that they are taking possession of the future site of their nest. Today, high speed photography allows a very precise analysis of the mechanism of the flight of insects. The rotatory motion plays a very important part in it. Their wings have some function of rotatory helixes. Often, when insects are attracted by the light of a lamp, they get closer to it describing helicoidal trajectories. Bees use a circular dance to communicate information on the nature and distance of a food source to exploit. We can go on: vultures reach higher altitudes by flying along helixes within rising columns of warm air, their trajectories are the shortest way up, given the incline; the torch fish uses a luminous organ to communicate, get food, defend himself, the organ immediately rotates into a black pocket the moment the animal decides it is time to 'turn off the lights'; and...while I am writing these lines, my fingers,my hand, my arm, they all move due to a combination of rotatory motions by different bone levers around their fulcrum.
Many examples in many fields of human activity demonstrate that rotatory motion is very common and very useful. Therefore it is very important to measure it. To rotate means to move of a certain angle in a certain time. Hence rotation measurements bring us to angle measurements. The angle that corresponds to a complete rotation around itself is called round angle. As an ancient tradition, the angle of measurement of an angle is the 360th part of a round angle, which is called degree. Rotations also multiple of 360 degrees, around points and symmetry axes, on the plane and in three-dimensional space, constitute one of the fundamental components of the figures and models illustrated in this book.

Giorgio Scarpa

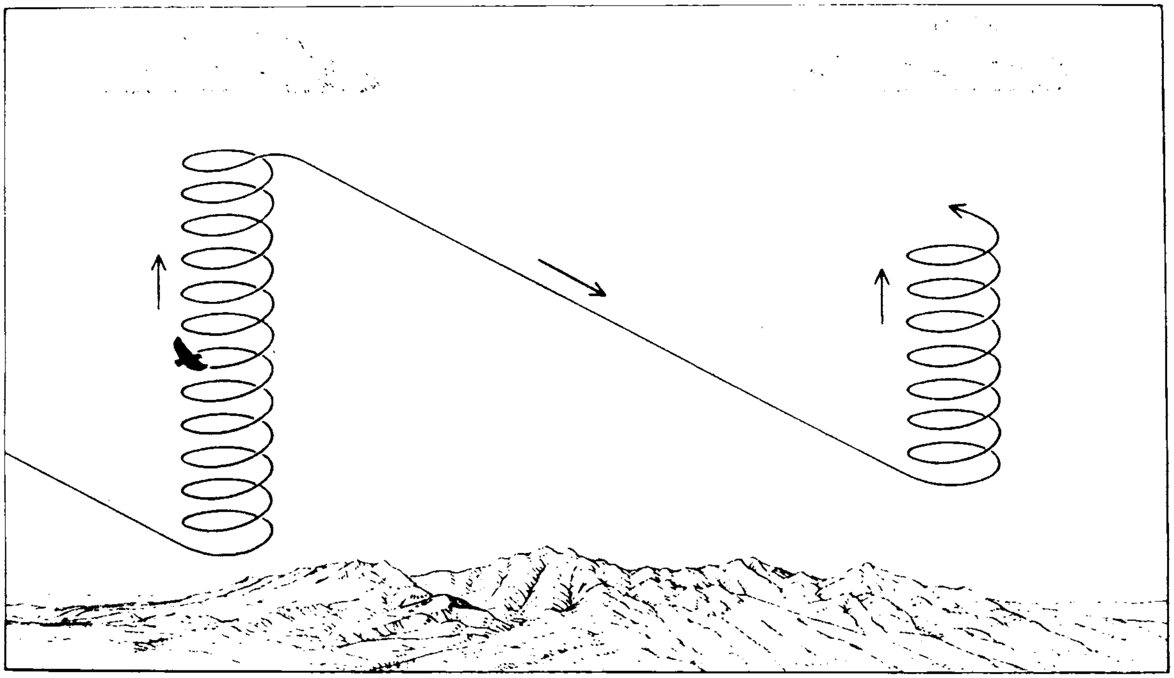
The spiral galaxy M81 in the Great Bear constellation.



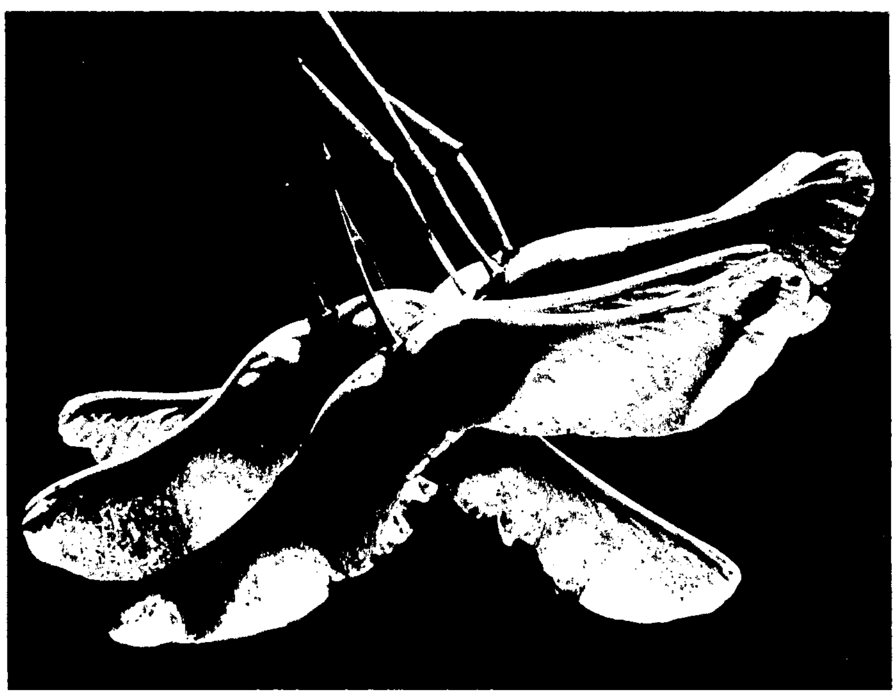
Reticular structures in rotation in a muscular fiber.



Circular dance of bees.

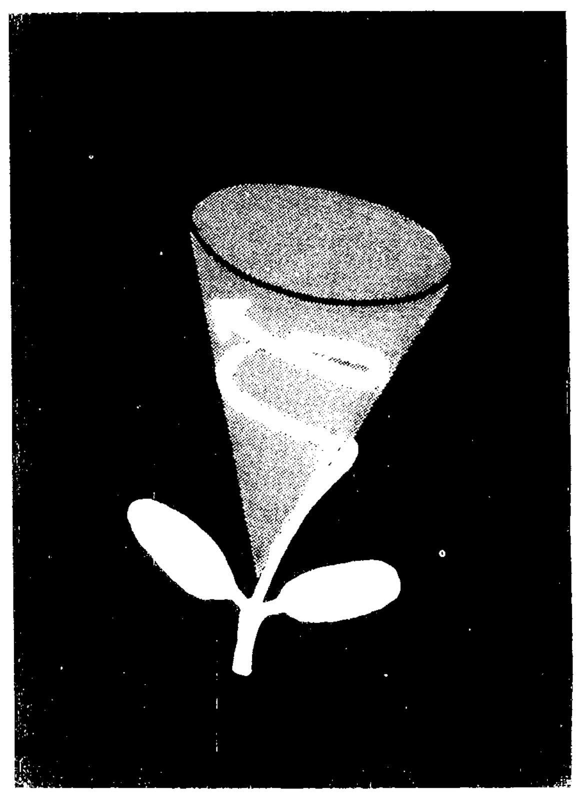


Tranfer flight trajectory of a vulture.



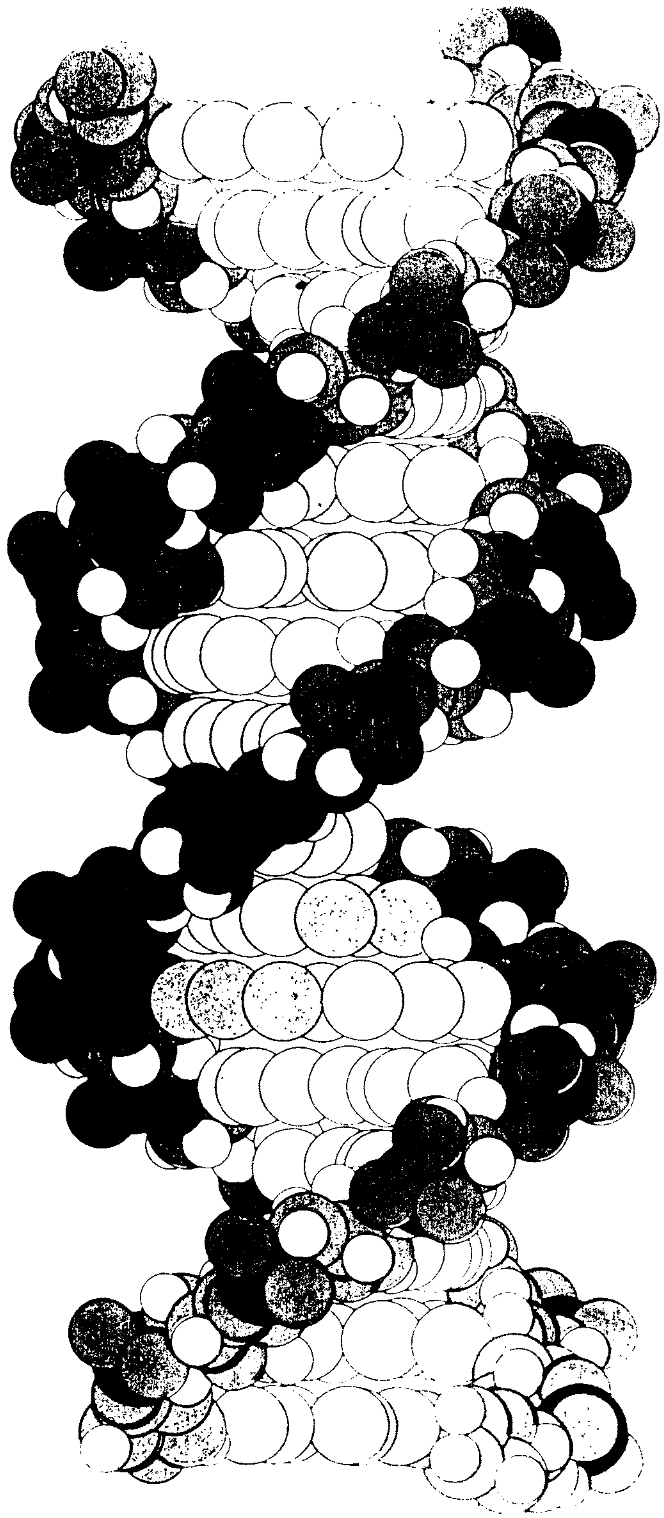
Maple tree seeds.

Rotating chemical reactions.



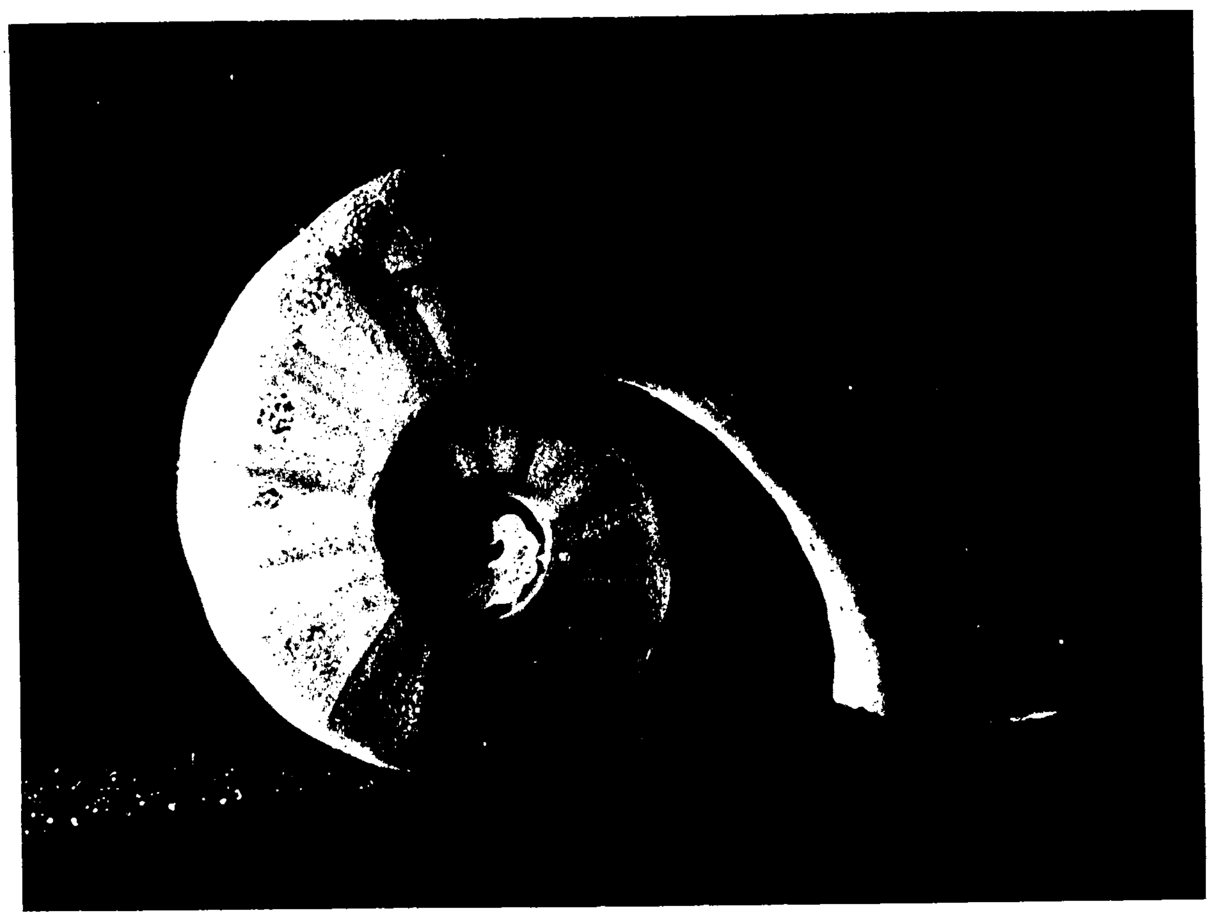
Circumnutation of a plant apex.

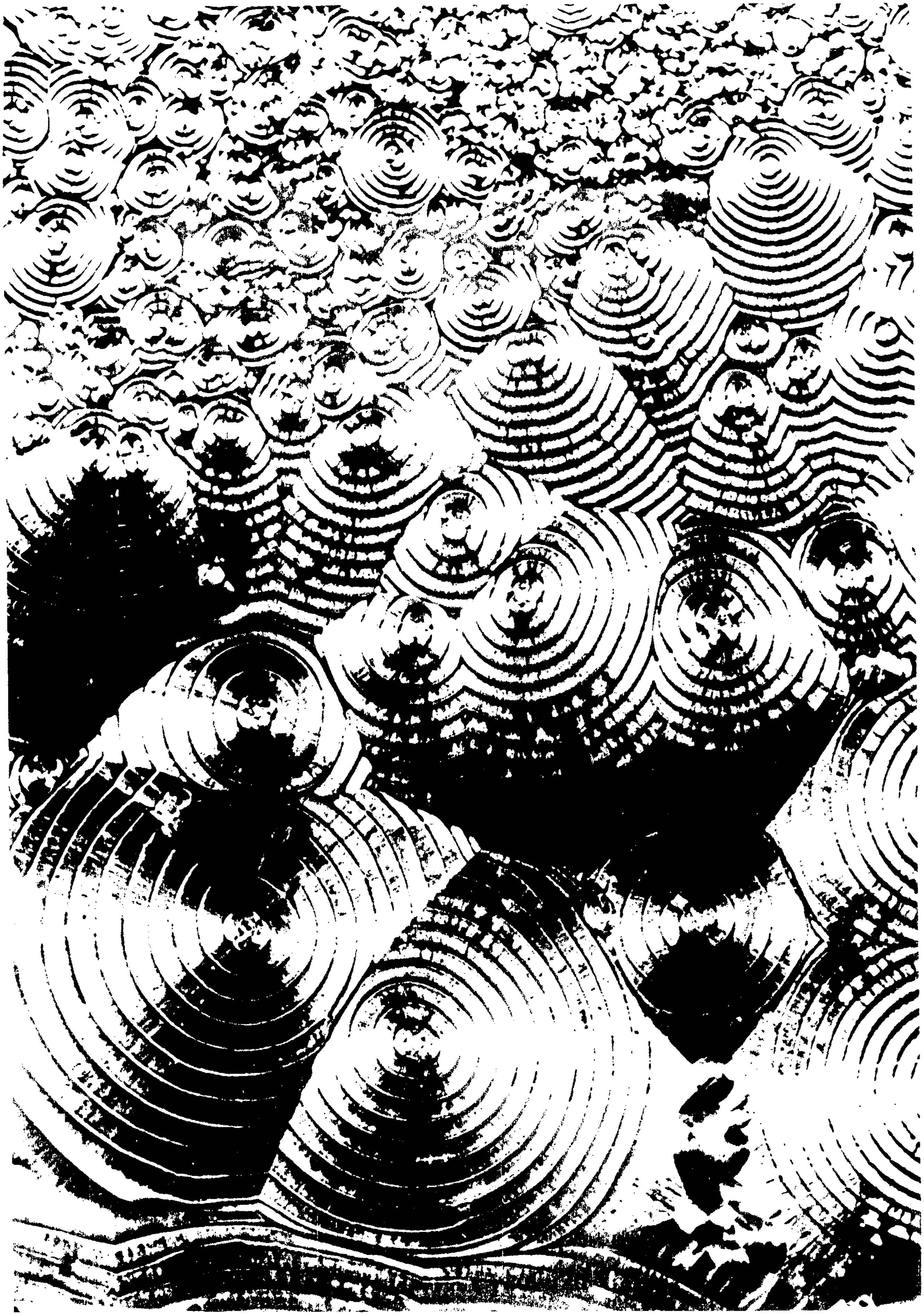
Model of DNA molecula.



Facing page: crystals of mono-cycle-esylamin.

Forms generated by rotation and expansion.

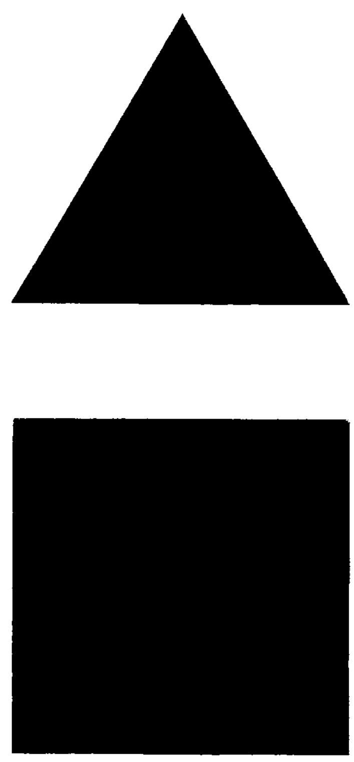




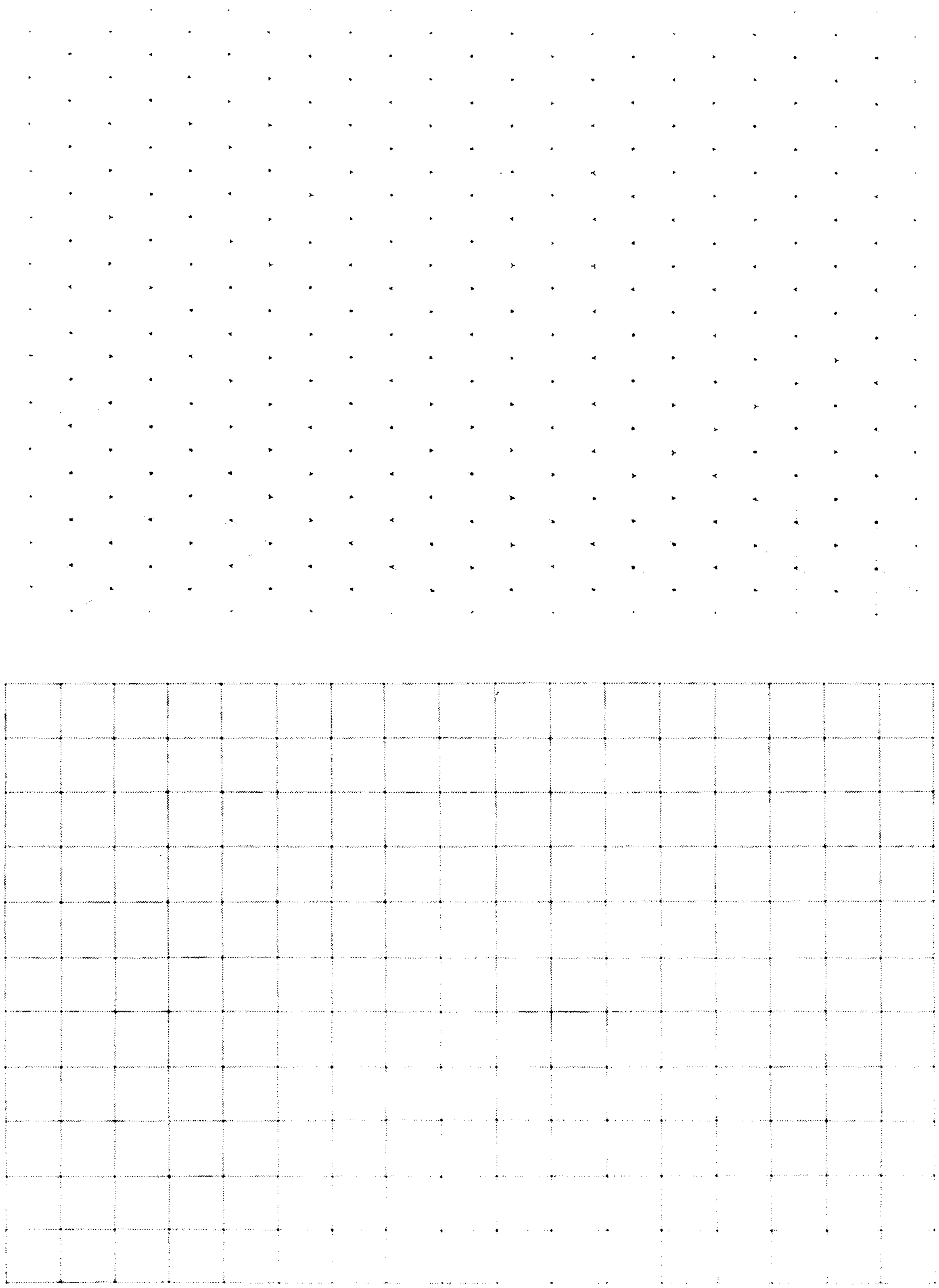
BEGINNING TO PROJECT COMPLEMENTARY MODULES

In nature, everything seems regulated by various degrees of structural and organizing complexity. A snow crystal, the parfume of a flower, are determined by certain properties of the molecular structure of the substance of which they are made. In the following examples regarding our investigation, the sectioning and recompositioning game of the two basic units refer to, as a guide on the plane and in space, the lines and nodes of triangular, square, tetrahedral and cubic reticula. One of the purposes of this study is to identify in which way and method or logical criteria, parts derived from the discomposition can be recombined on the plane to form configurations in which their three-dimensional fold-outs originate repeatable and invariable modules, and how to proceed in their organization and spatial coordination. For this purpose a reference point is given by the five Plato polyhedra solids, the first and simplest group of the vast family of polyhedra.

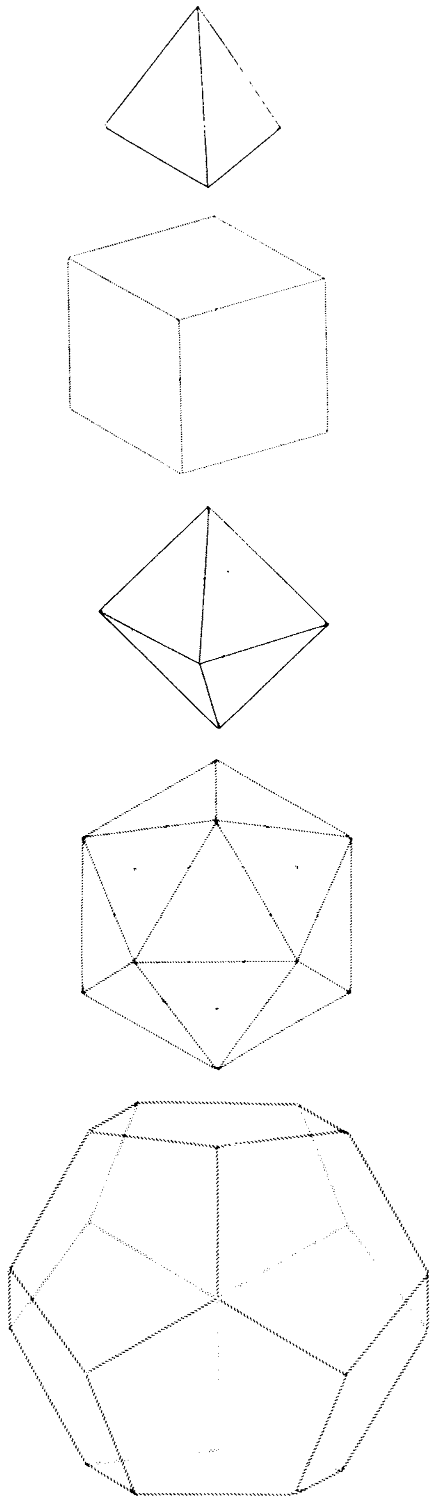
A connection between plane and three-dimensional configurations can be made through the study of the square and the equilateral triangle. Each of these geometric units can be discomposed in a limited number of parts. The variety of the discompositions is, on the contrary, relatively infinite. We find them at the beginning of a process of formal and coherent tranformations, which represent the object of this study. The simplest and most elementary subdivision of a plane surface into geometric modules of the same type is given by configurations of equilateral triangles and squares. Three-dimensional schema based on these two figures seem to frequently reflect the spatial arrangement of the particles that give structure to matter.



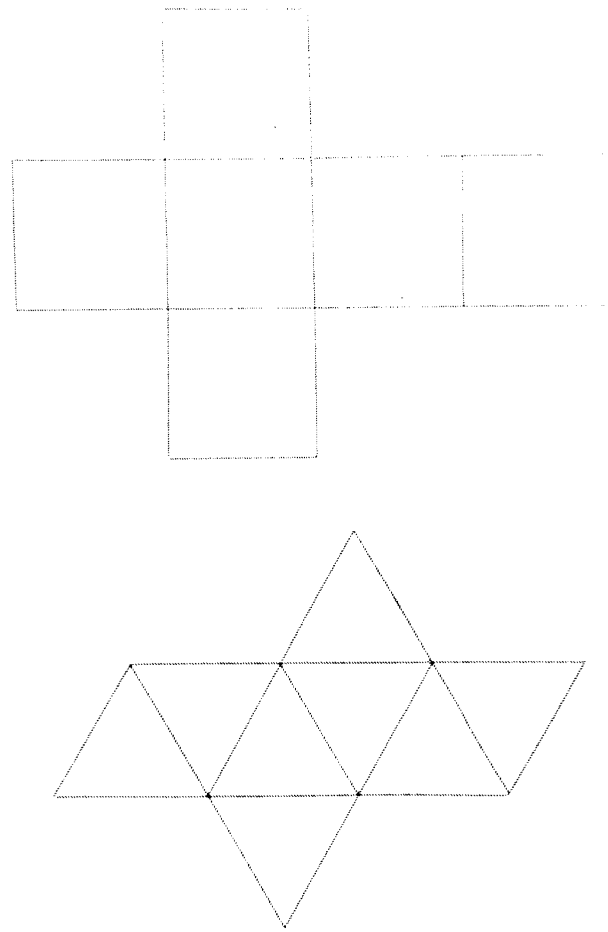
The two basic forms.



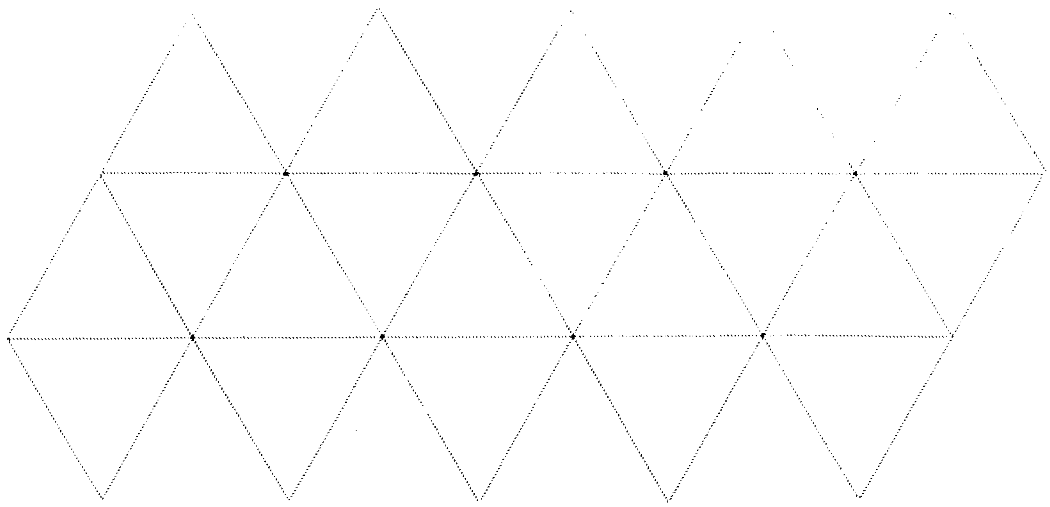
The two spatial reticula.

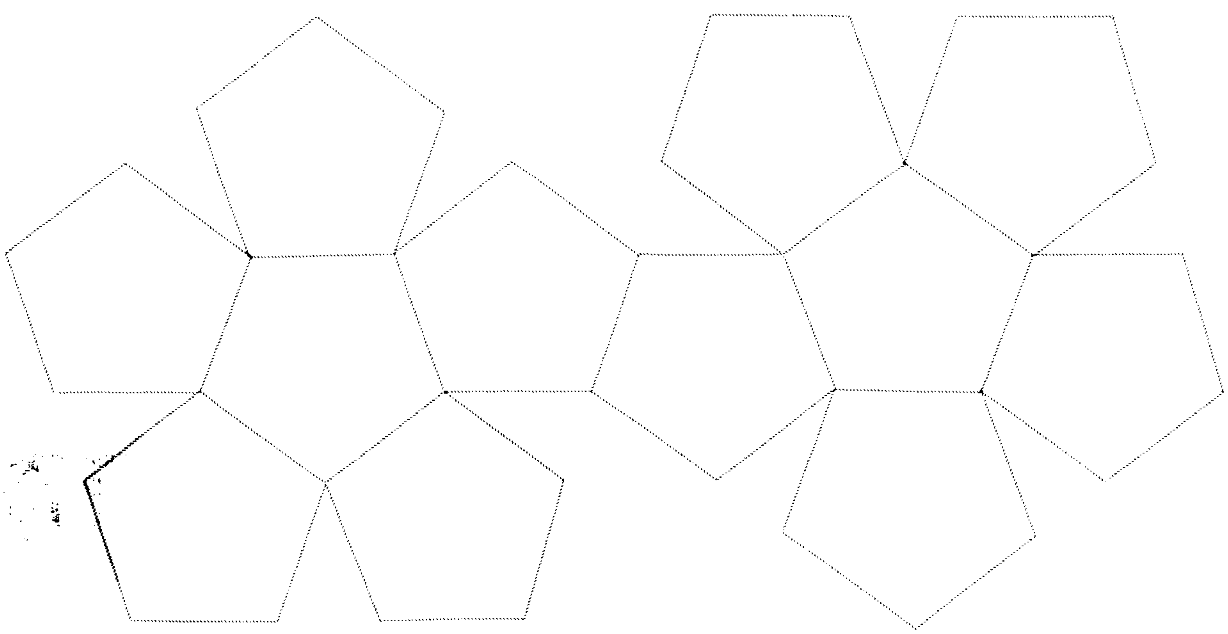


The five Plato polyedrons.



The plane fold-out of the hexahedron and the octahedron.





The plane fold-out of the icosahedron and the dodecahedron.

THE USE OF RETICULA

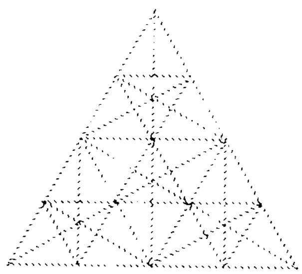
We will begin by examining some bi-dimensional reticula, their structure and the organization of form on them.
With families of squares of different sizes we can more or less densely populate a given plane surface. Starting with a square of given dimensions in which we will divide the sides in equal parts by tracing parallel lines to them, perpendicular between themselves, starting on the division points. For example, dividing each side of an 8-cm square in progressively two, four, eight parts, we obtain 4, 16, 64 smaller squares (submultiples) in which the sides respectively measure 4, 2, 1 cm. We also obtain reticula which become more and more dense.

In the same way we will construct the equilateral triangle reticulum, dividing the chosen equilateral triangle in half. Connecting the median points with straight lines that divide the basic equilateral triangle in four submultiple equilateral triangles, until defining the modular structure according to the desired degree of refinement.

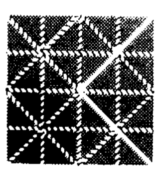
The lines which constitute the reticulum cross each other at points and nodes that are all of 90 degrees in a square reticulum and all of 60 degrees in a equilateral triangle reticulum. The growth of the families of square and triangular submultiples proceeds with the same numeric increments, 4, 16, 64... and so on.

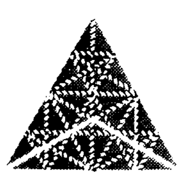
These regular configurations that cover the entire square or equilateral triangle plane are the simplest and most elementary modulated structurings, but obviously not the only structures capable of formally characterizing, through points, lines and regions, the surface of the two basic figures. Other examples of structures are created by combinations of orthogonal and diagonal lines for the square, and by straight lines connecting the vertexes with the median points of the sides and lines parallel to the sides for the equilateral triangle.
Let us stop here for a moment to see how we can use these structures, keeping in mind that the choice of a reticulum has to relate to the type of experiment that we want to conduct.

Once we have traced a reticulum of of perpendicular and diagonal lines in the square, we will section it in the simplest possible way: we will divide it in two parts, we will do the same with the equilateral triangle. For example, like in the figures shown below.

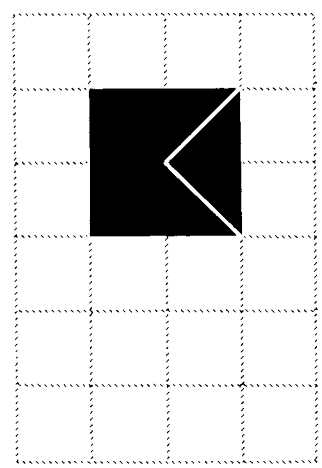


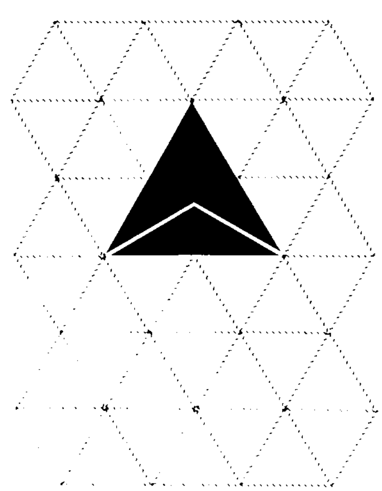
We will then put the two figures derived from the section of the square in a structure constituted of square modules, and the forms derivated from the section of the equilateral triangle, in a structure constituted of equilateral triangles. The forms will be cut out of paper board and we will put them on the modulated surface. The sides of the figures must be the same or a multiple of the basic module that forms the structure.





Vertexes and perimeter of a form must coincide with the nodes and the lines of the reticulum.
We will now worry to define in which way to move those forms on the structured plane. Then to see what possibilities of combinations exist between them when their positions change on the modulated surface. A symmetry operation that can be applied, among the others and without modifing that form, is rotation.

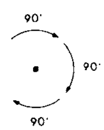




We will examine only the rotation movements of the forms included in in the following cases:

1. a form can turn around a center of rotation within the form itself or can be located on the vertexes or the sides that constitute its perimeter;

2. a form car rotate one or more times around one or more centers of rotation, clockwise or counter-clockwise, describing angles which are multiple of 360°;



3. a form can be rotated around a center of rotation describing various times a same angle multiple of 360°;

4. we can rotate two or more forms simultaneously around the same center of rotation;

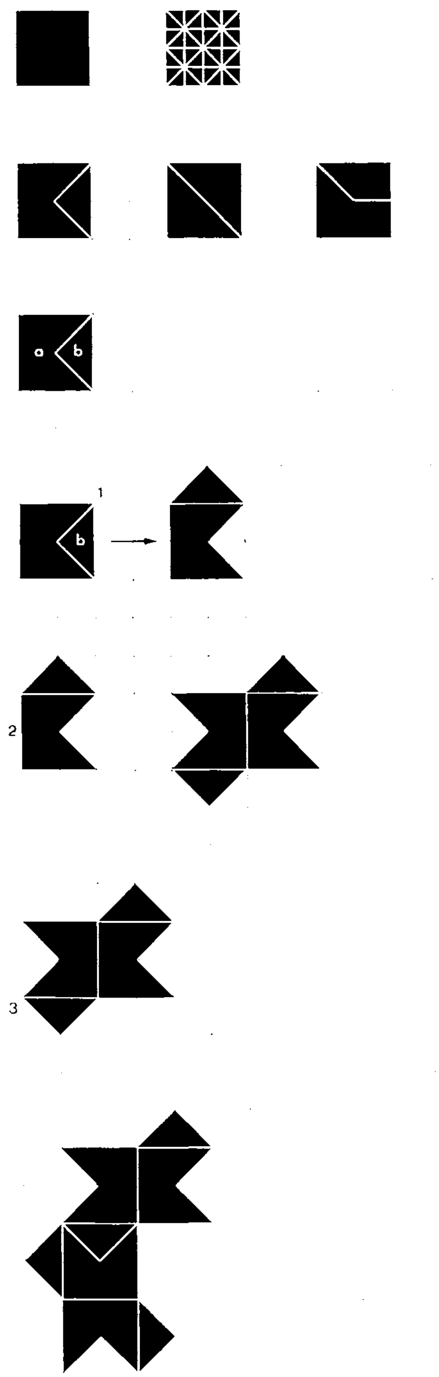
5. a form *a* can be rotated around a form *b* and viceversa, when both have a common center of rotation;

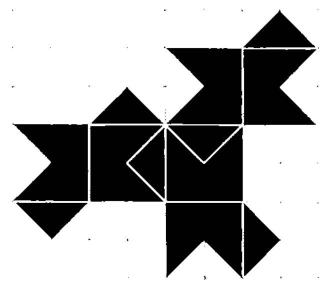
6. we can add a form sitting in a determined position on the plane with the area to the same rotated form.

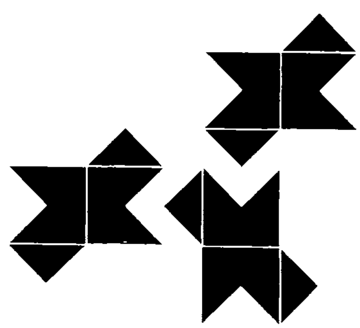
The forms combined in rotatory plane symmetry always result to be in contact with each other in their final configurations, and never overlapped. The operations of grouping and organization of the forms help understand how they combine themselves on the modulated surface, not according to a casual relationship, but in such ways that define their reciprocal interdependence, in relation to the construction of modular plane fold-outs, which, folded in third dimension, will originate three-dimensional modules, fractions of the global volume of polyhedrons.

After having traced on the square a reticulum constituted of perpendicular and parallel lines, having chosen a sectioning itinerary among the many which are possible, having sectioned the square in two parts; let us look now at the operations to be done in accordance to an ordered logical sequence: rotate *b* clockwise of 90°, pivot in 1 (passing over form *a*). Rotate the resulting figure of 180° about 2. Pivot in 3, rotate two times the newly composed group of four parts.

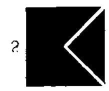
The entire hexahedral plane surface (external fold-out) is defined by 4 rotations around 3 centers of symmetry, and is articulated in 3 distinct fold-out plane groups, each being constituted by two squares (4 parts). The first rotation tranforms the bisected square, which has a specular symmetry, in a new figure that is no longer symmetric. Starting from this sequence, all the following ones show operations of cyclic rotatory symmetry.



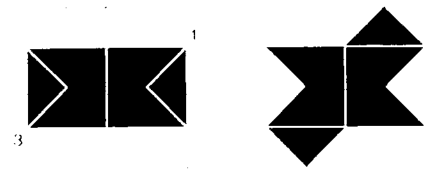




By changing the order of the rotations around the same symmetry centers, the final result is the same, while the intermediate formal conbinations of $a + b$ do change.



1st example: center in 2, rotate $a + b$ of 180°, then rotate the two triangles b of 90°;



2nd example: rotate $ab + ab$ of 90° two times with center in 1 and all the triangles b of 90° about the vertexes of each square (clockwise rotations).
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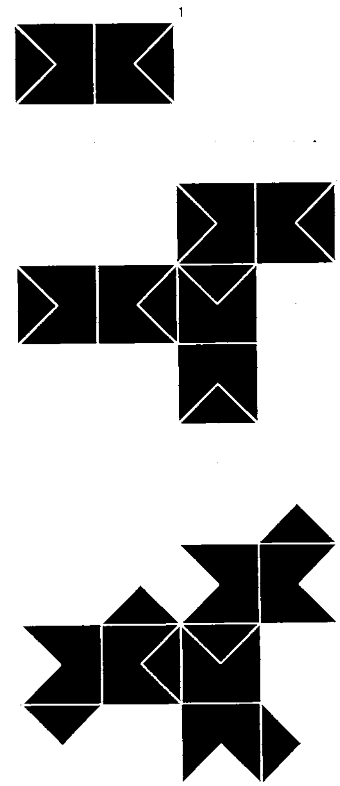
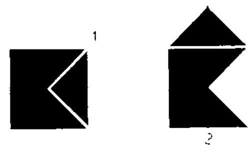
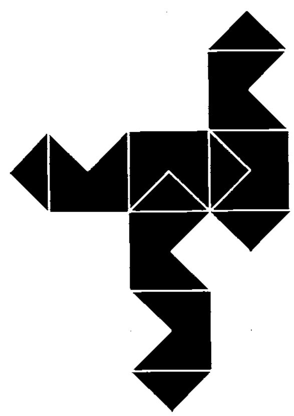


We can obtain a new fold-out with the rotation of *b* clockwise around 1 and of *a* + *b* around 2,

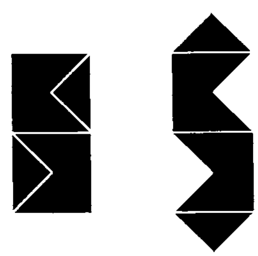


then, two counter clockwise rotations of 90° with center in 3

define the hexahedric configuration of three distindt fold-outs.

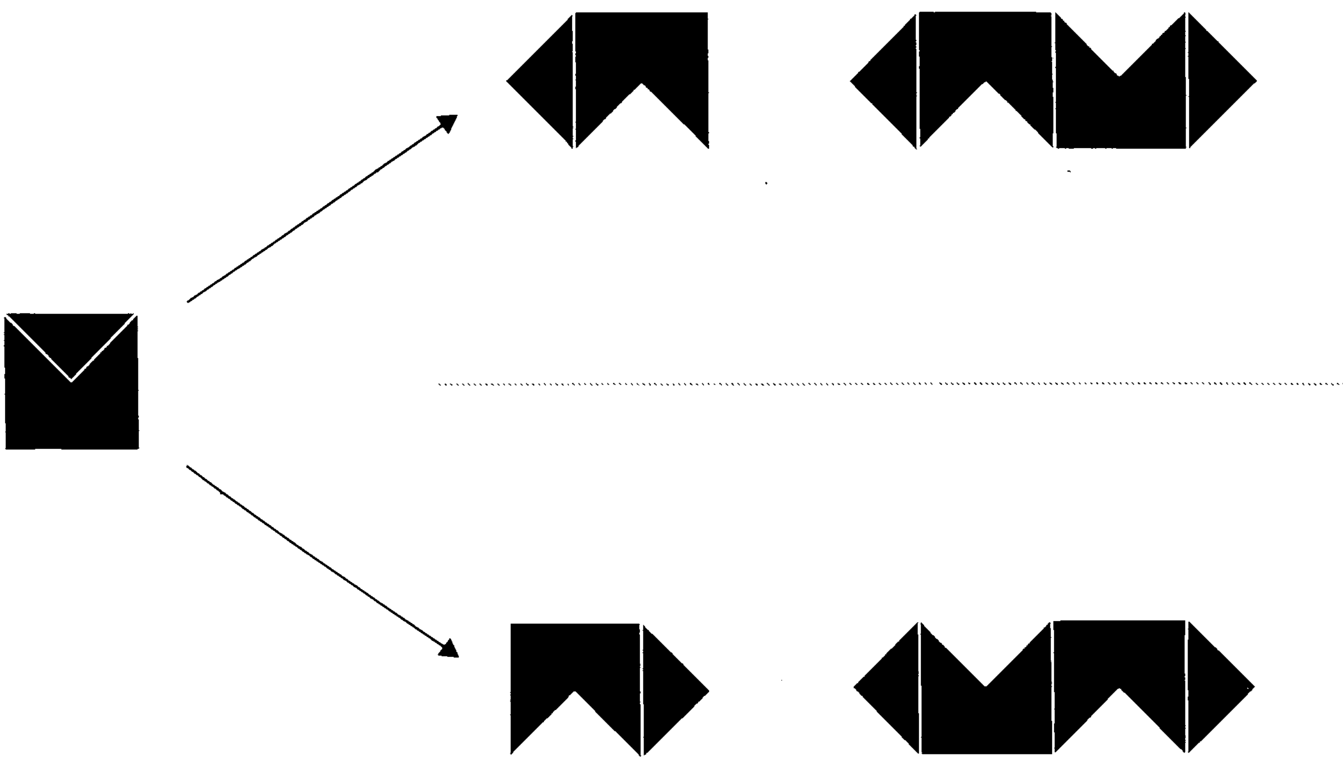


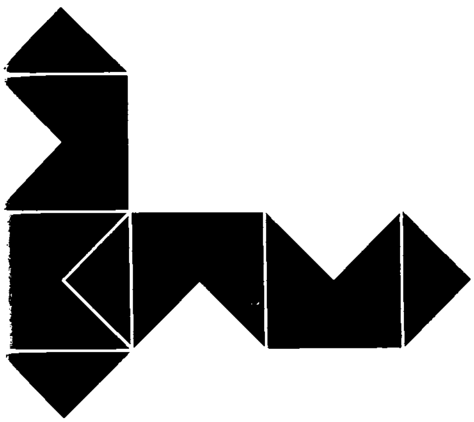
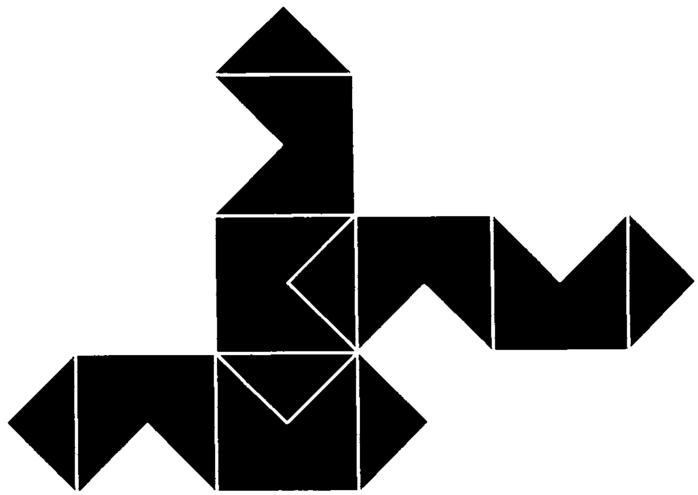
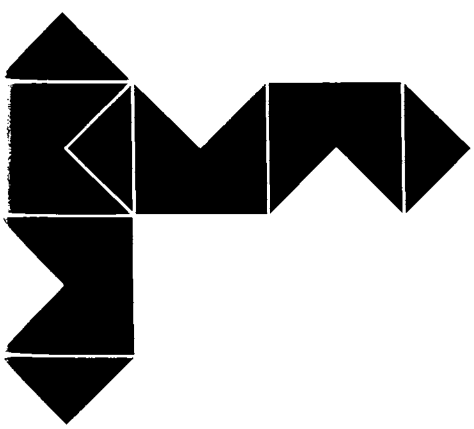
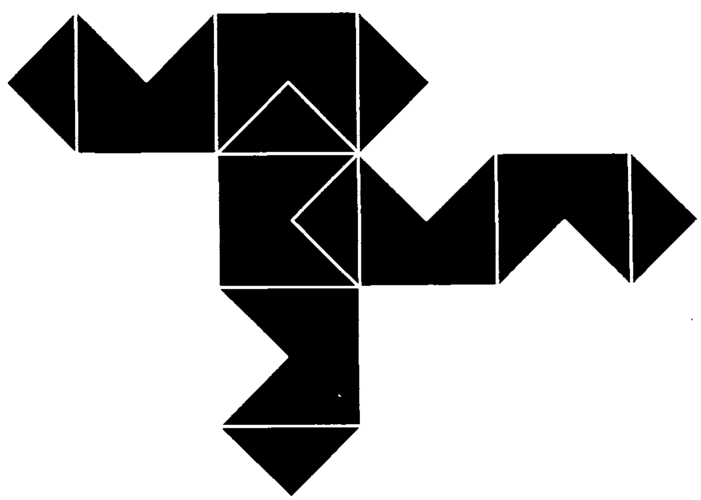
a different combination of *a* + *b* , similar to the previous one, originate the four-part group (fold-out) if we rotate the triangles *b* of 90°.



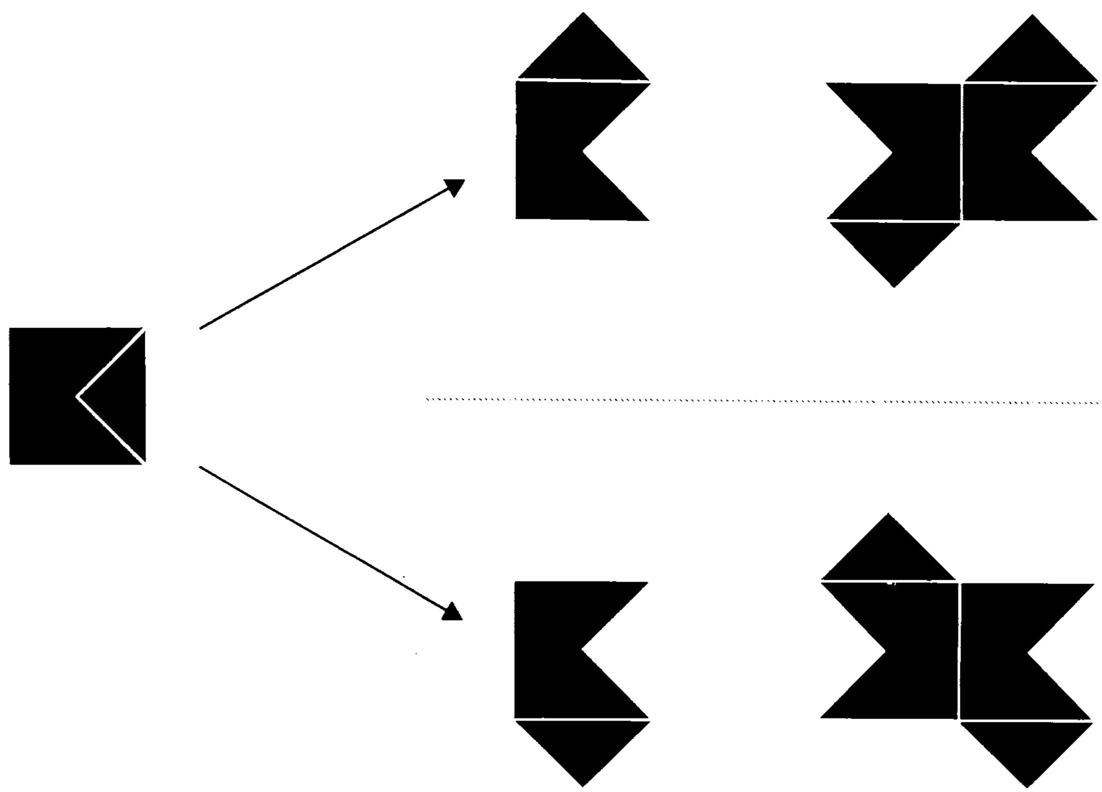
From this section of the square derive four different hexahedric fold-outs, of which two can be determined through an inversion of the rotatory movement, or by reflection of the plane of symmetry.

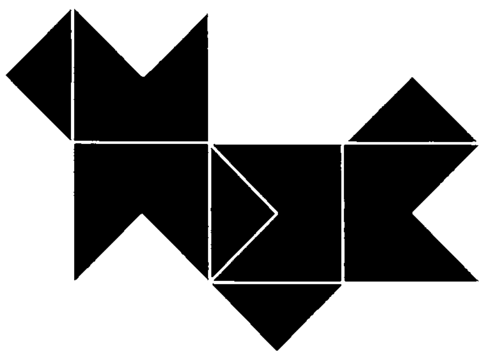
Overview example with eight configurations in fold-out progression, laid out in two sequences, one to the right and one to the left.

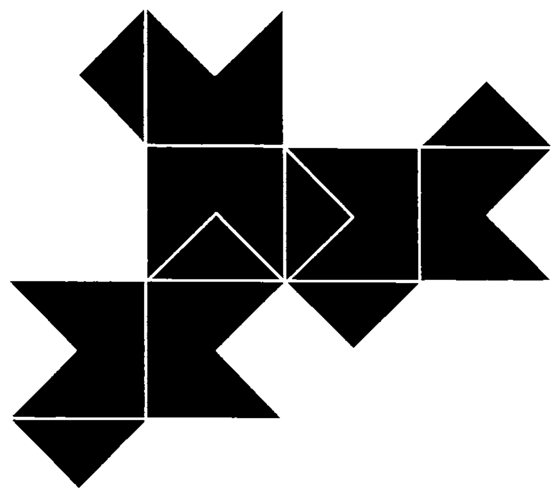


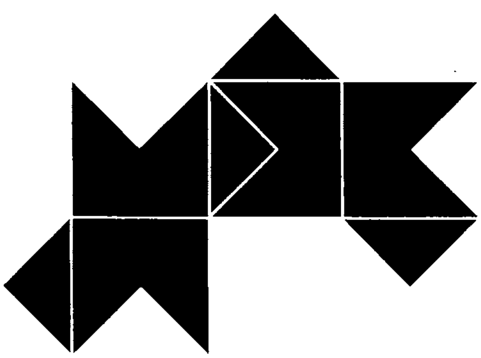





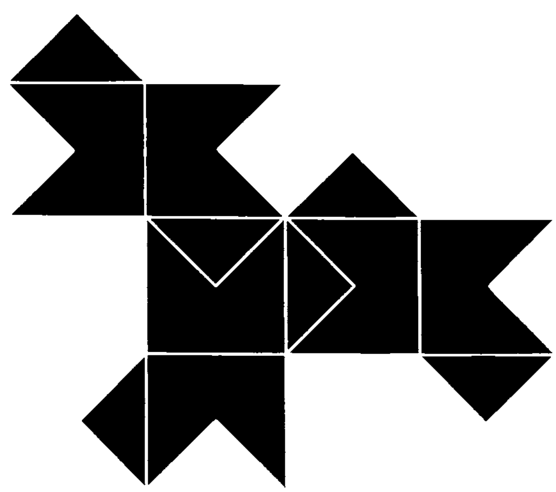
Another example similar to the previous page.

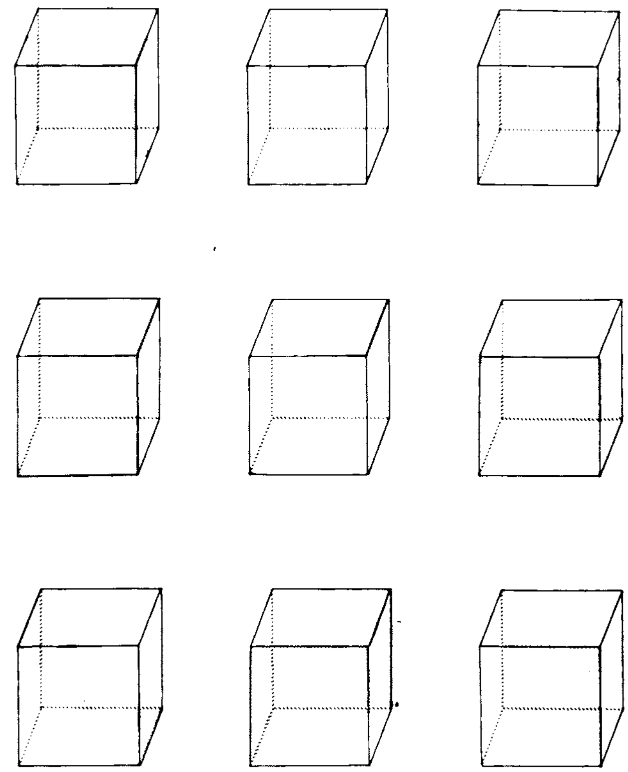








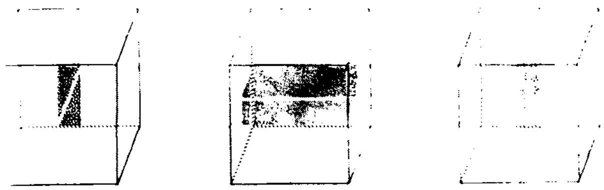




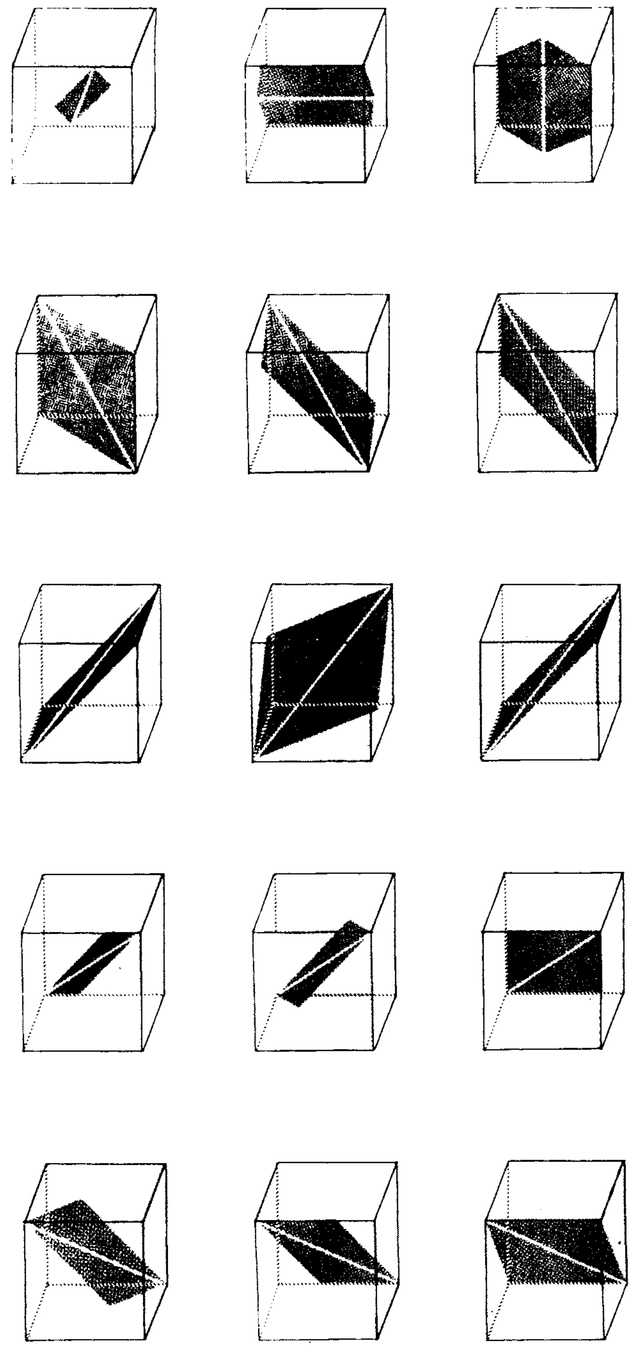
PLANES AND DIRECTRICIES OF THE CUBIC SPACE

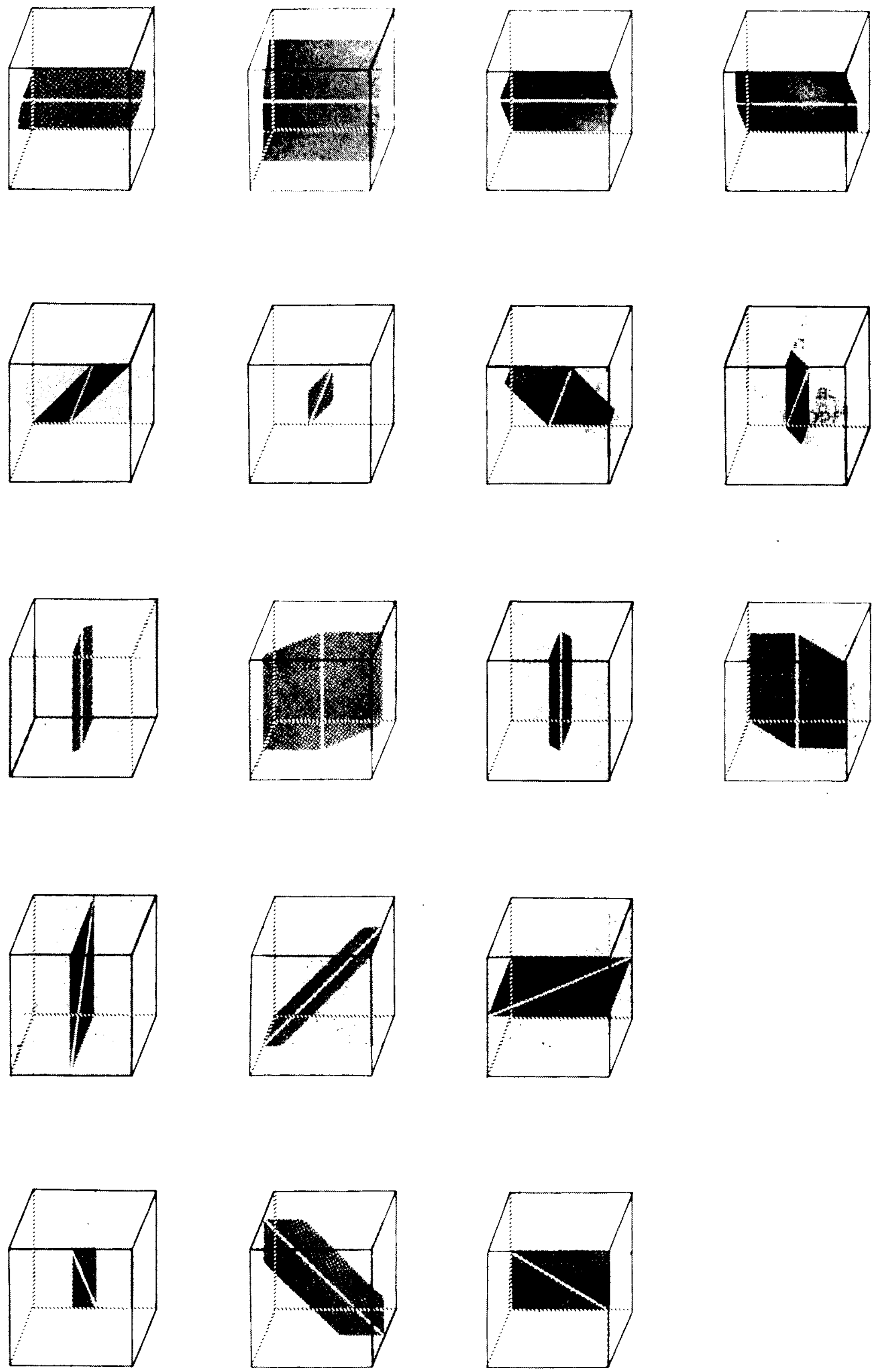
A solid's surface can be considered formed by plane figures (the equilateral triangle as face of the tetrahedron, the square as face of the cube, etc.) in the same way that its inside can be seen as constituted by plane figures resulted from sections. We will begin by exploring the cube, starting with a section that will devide it in two rectangular parallelepipeds of equivalent area and volume. This section is a square identical to the six faces that form the cubic surface. We can divide the cube in this way only three times. The three sectioning square planes, perpendicular to each other, are arranged in three different spatial orientation, like the faces of the cube. The plane that devides the cube into two prisms with a triangular base is a rectangle whose sides are the diagonal of two faces and two edges of the cube. We can visualize six of these planes inside the cube, each with a different spatial orientation.

If we intersect one of the three square planes with each one of the other two in the way shown in the example, we can see that the possible intersections are three: they happen to be along three lines that constitute the connecting straight lines between the central points of the the six faces of the hexahedron.

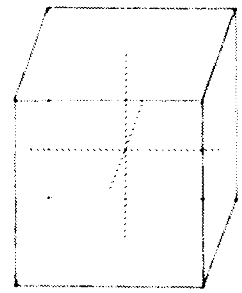


Instead, if we combine the six rectangular planes and visualize all the positions that each one can take in regards to each other, we find that the possible intersections are fifteen: only three times a plane intersects another plane along the three straight lines connecting the centers of the faces of the cube; and three times can a plane intersect another one along each of the four diagonals that unite in pairs the opposite apexes of the cube.

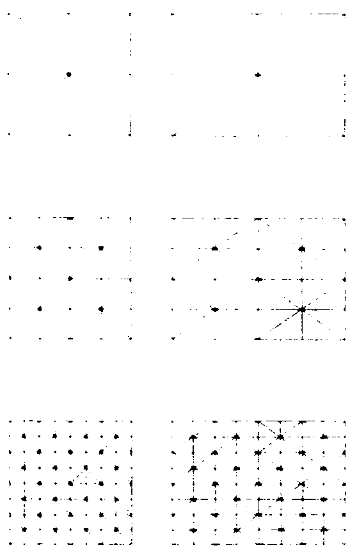


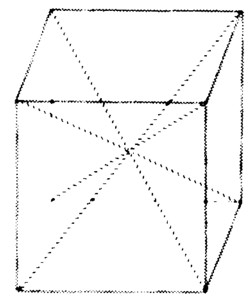


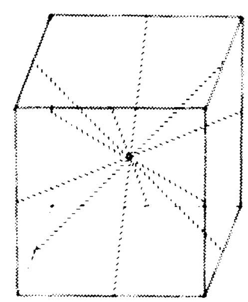
The intersections between square and rectangular planes are in all eighteen: a square plane and arectangular plane intersect four times along each ternary directrix, and only once along an intersection line equal in lenght to the diagonal of a face of the cube. Since the rectangular plane can assume six different positions within the cube, the square plane intersects it six times, assuming two times the same position in the sense of each ternary direction.



As we see in these examples, the intersection lines of the planes are at the same time connecting straight lines between the six centers of the faces, the eight apexes, the twelve edges of the cube.



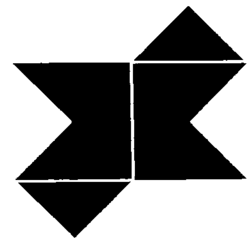




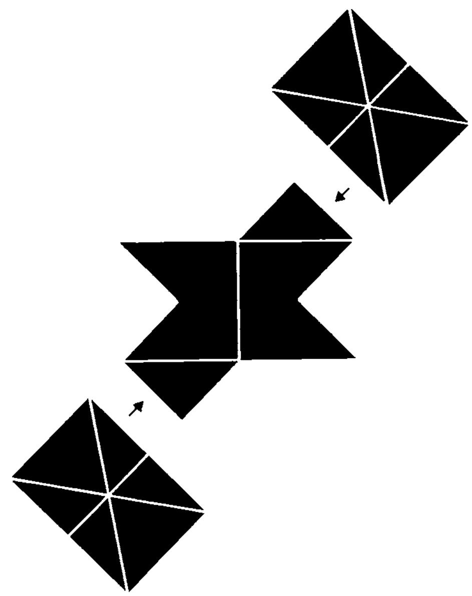
Moreover, they can assume the function of lines of the bi-dimensional structure of the square (internal surface plane), and of the rectangle, internal plane of the cube.

In this way we also establish a precise relationship of geometric correspondence between the two strucural grids whose refinement can be more or less accentuated as shown above: to a grid with lines and points more and more dense belonging to a surface plane-figure (square) corrispond grids with the same number of lines and points of an inside plane-figure (rectangle). The relations that unite the inner planes with the squares (or parts of them) that constitute the surface of the cube are infinite.

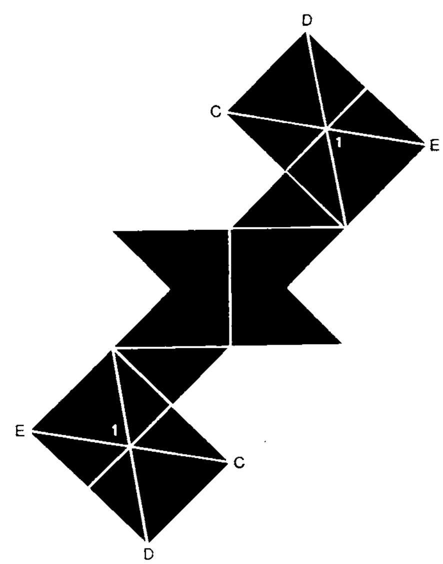
Let's see now how we can connect the internal planes with the hexahedral surface fold-outs:

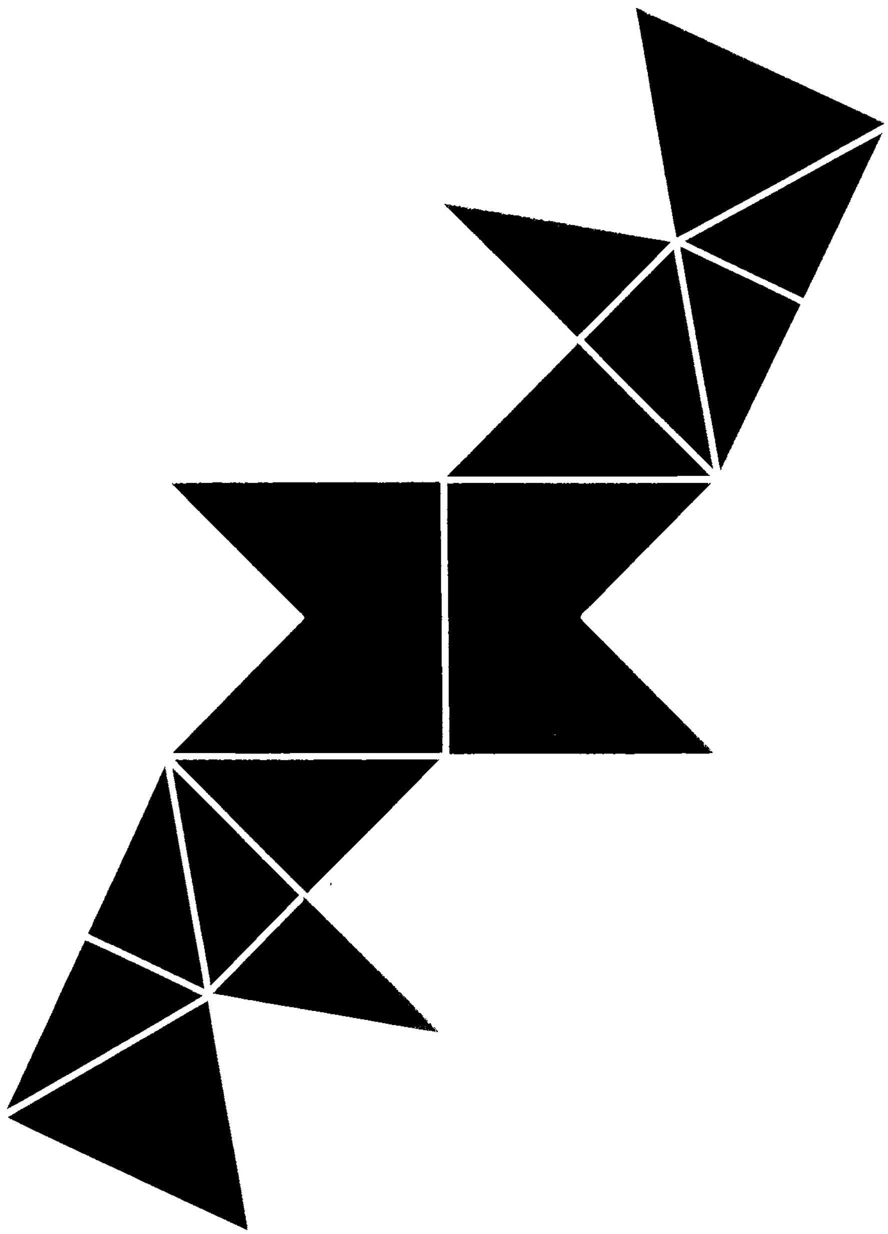


1. join two $1 \cdot \sqrt{2}$ rectangles to one of the external hexahedric surface fold-outs, such that half of their longer side results adjacent to one of the two free sides of triangle *b*.

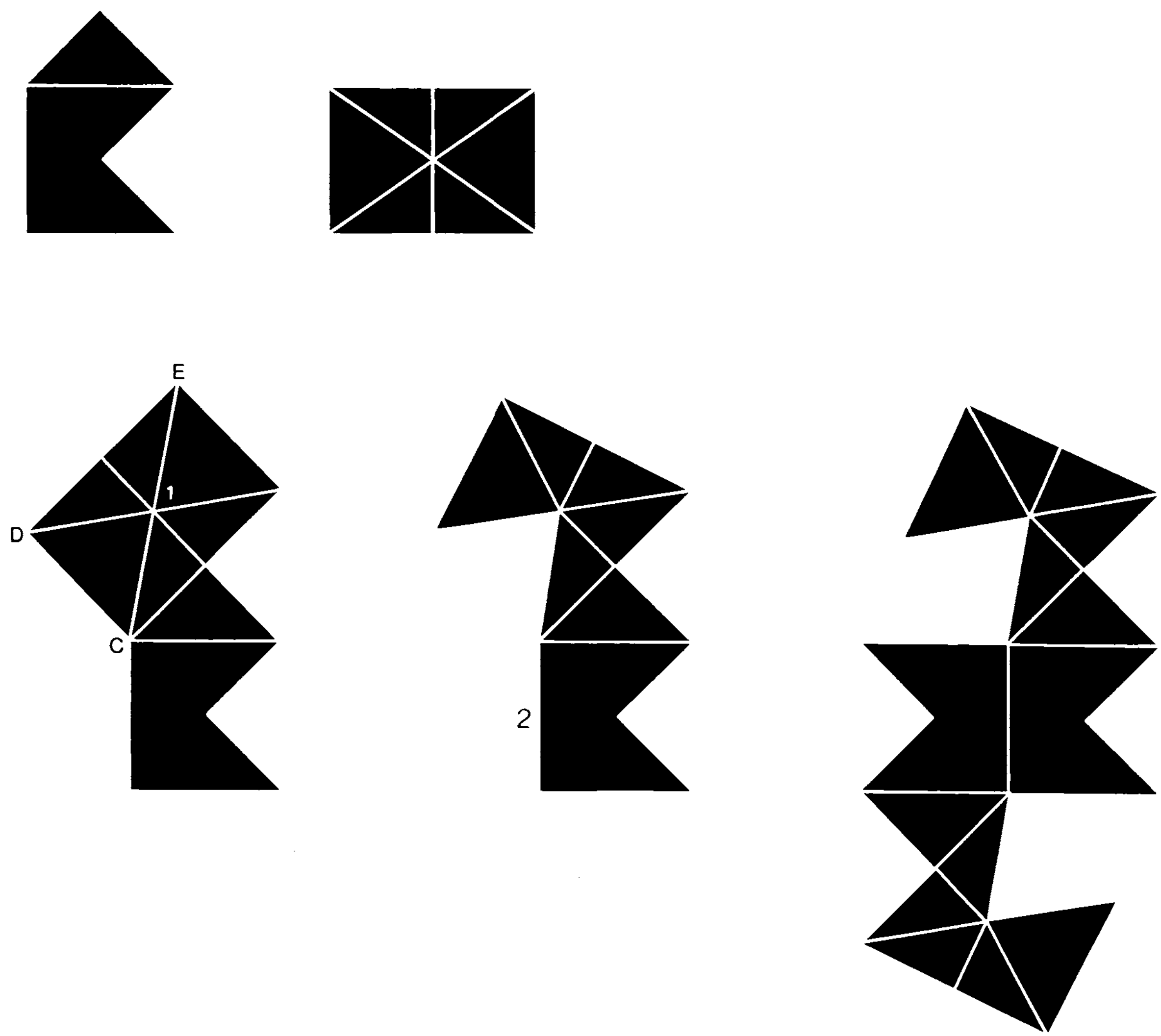


2. rotate the rectangular triangles C-D-E of 70° around points 1.

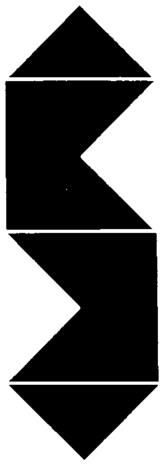


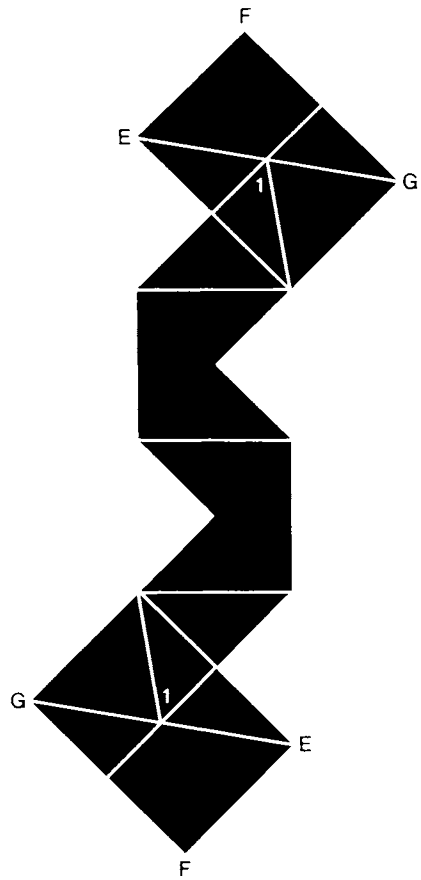


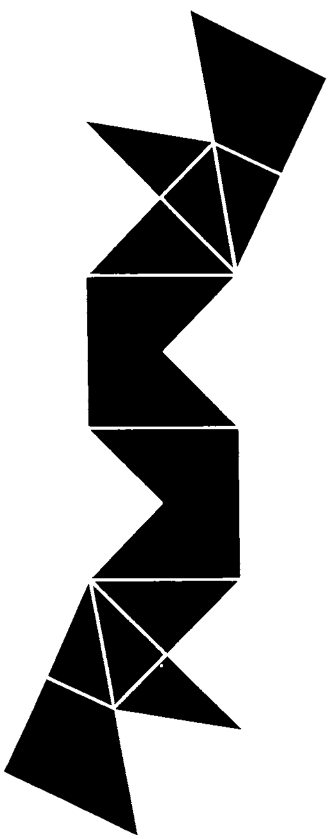
This configuration, articulated in 14 parts (four derived from the square module and 10 from the $1\cdot\sqrt{2}$ module), is one of the three hexahedral surface fold-outs capable, if folded, of exactly occupying a third of the entire cubic cell.



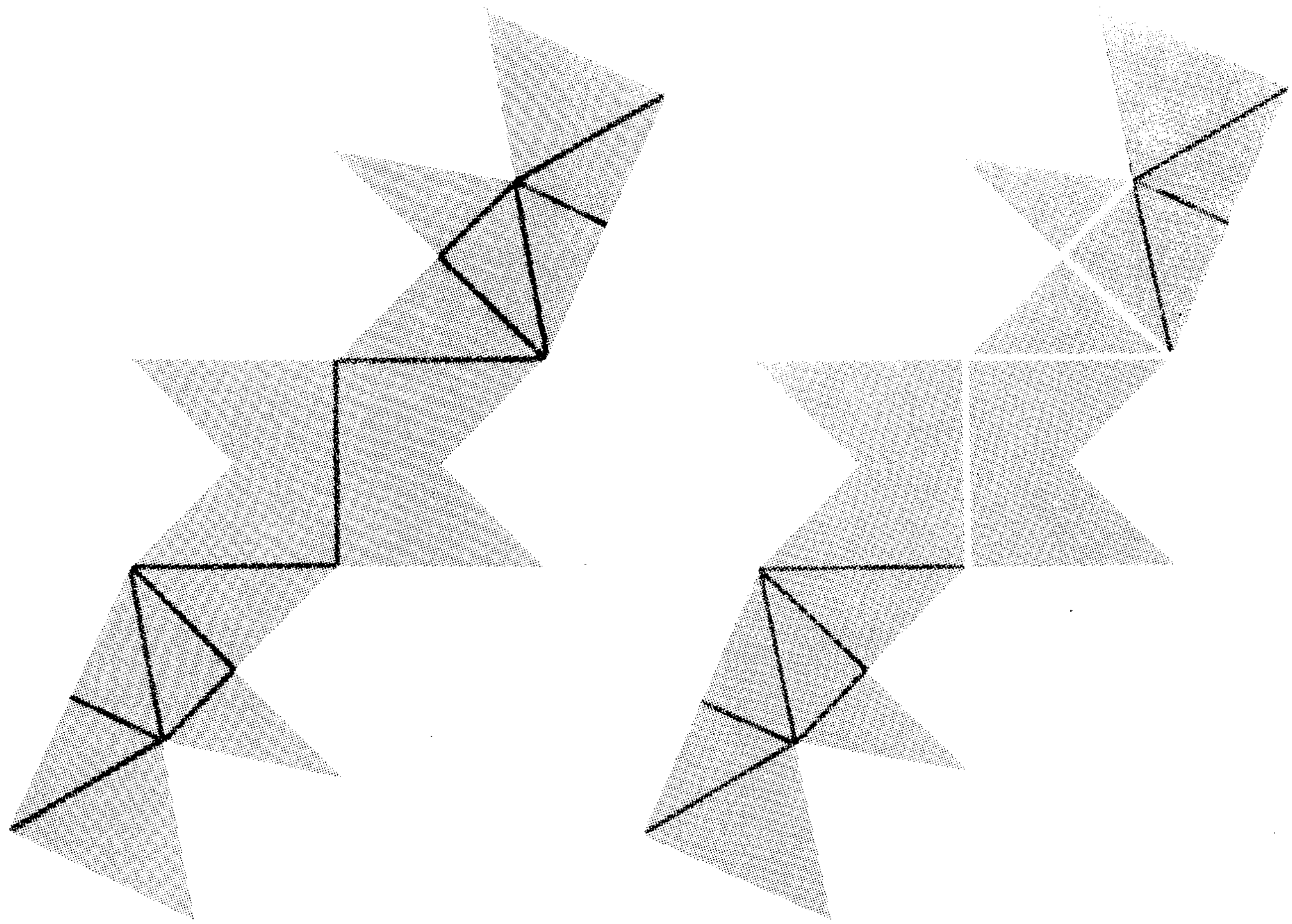
A different conbination of the modules originates a varied articulation of the same fold-out, third part of a cube, when we connect the $1 \cdot \sqrt{2}$ rectangle to *ab*, following up with a rotation of 70° around point 1 of rectangle C-D-E, and another of 180° around point 2.







In this case, by repeating the known joining operations, with a clockwise rotation of 70° of the E-F-G triangles, center in 1, we obtain a new fold-out which surface will delimit again a third of the cube's volume, this time in a different way from the previous ones.



ROTATION AND FOLDINGS OF THE FOLD–OUTS

The connections of the external surface fold-outs with the internal planes of the hexahedron determine, as we saw before, new configurations, new fold-outs, and by folding them, therefore by going from plane to space, we obtain three-dimensional modules. The transition from two dimensions to three dimensions depends on:

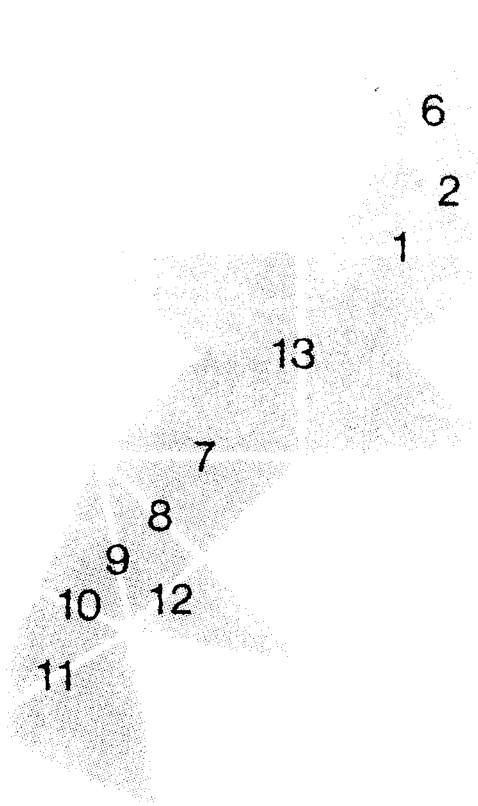
1. making the lines that dimension the fold-out surfaces assume the function of rotatory axes;

2. outlining folding sequences along these linear configurations, that constitute determined paths;

3. the choice of the more logic and economic paths;

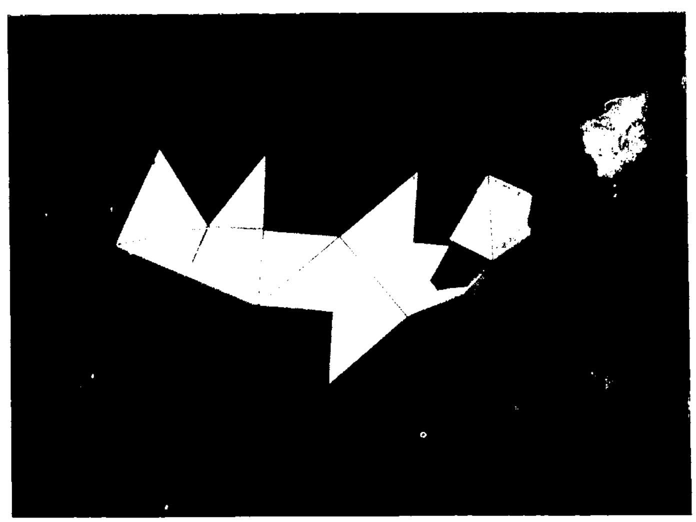
4. rotating (folding) according to determined angles, parts of the model around these lines (axes);

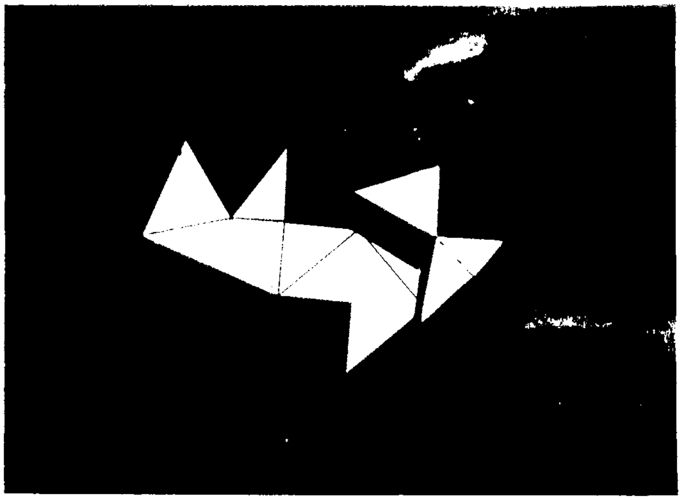
5. regulating the number and type of operations, in regards to schema of formal organization.



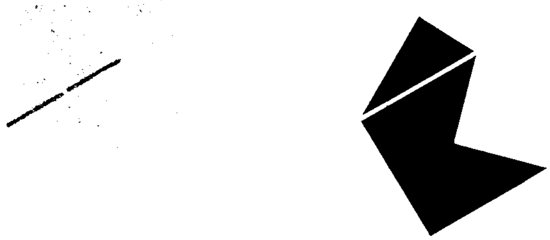
cubic cell to which we will refer the folding sequences of the model.

Examples:
folding order indicated on the internal lines by the numbers 1-13, in relation to the definition of the module;

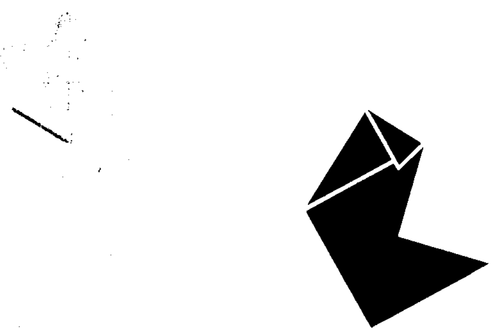




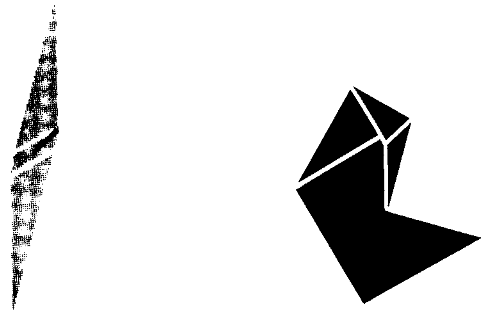
The linear path around which to rotate the planes of the model is the following:



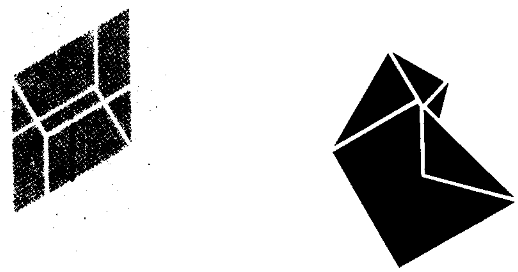
1. edge of the cube



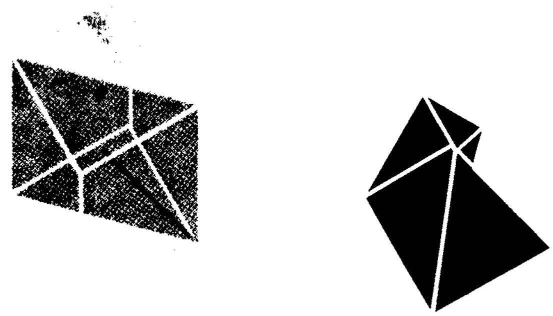
2. half diagonal of a face



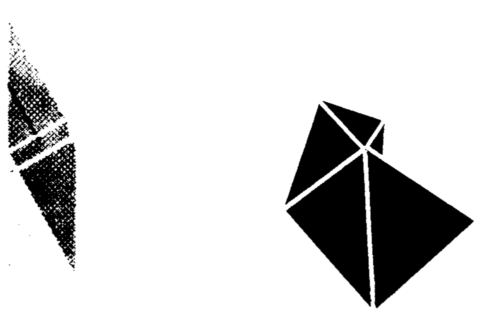
3. internal half diagonal



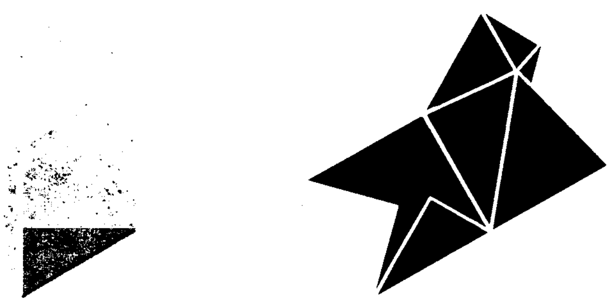
4. internal half axis



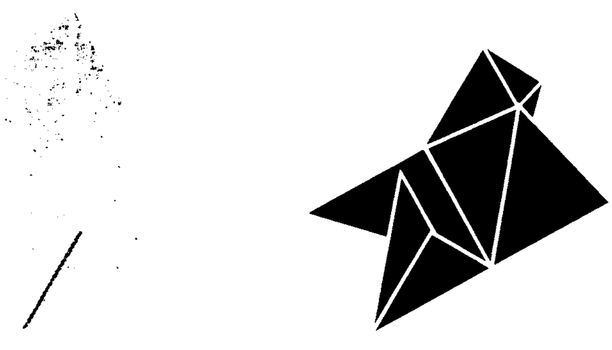
5. internal half diagonal



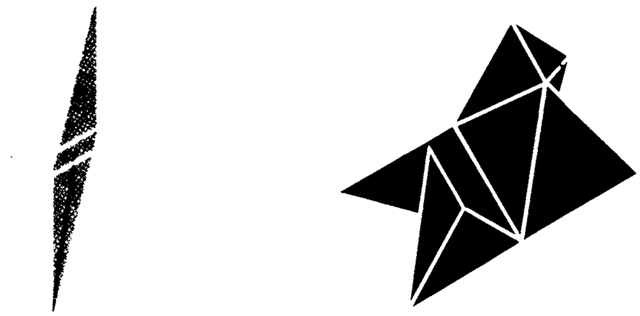
6. half internal axis



7. edge

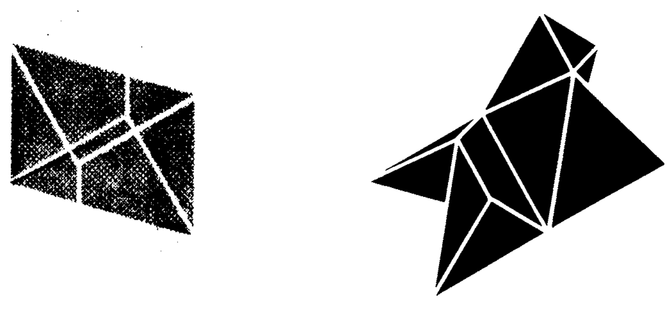


8. half diagonal of a side

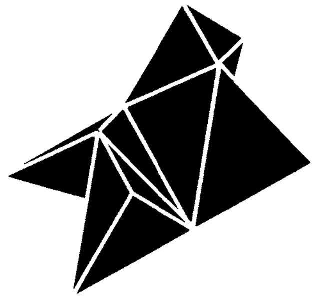


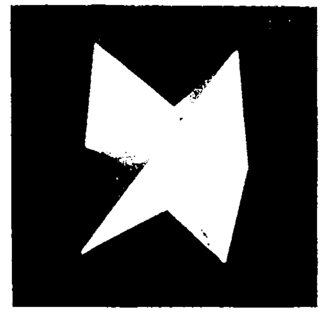
9. half internal diagonal

10. half internal axis

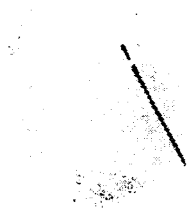


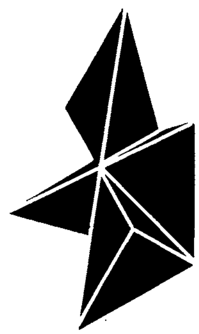
11. half internal diagonal



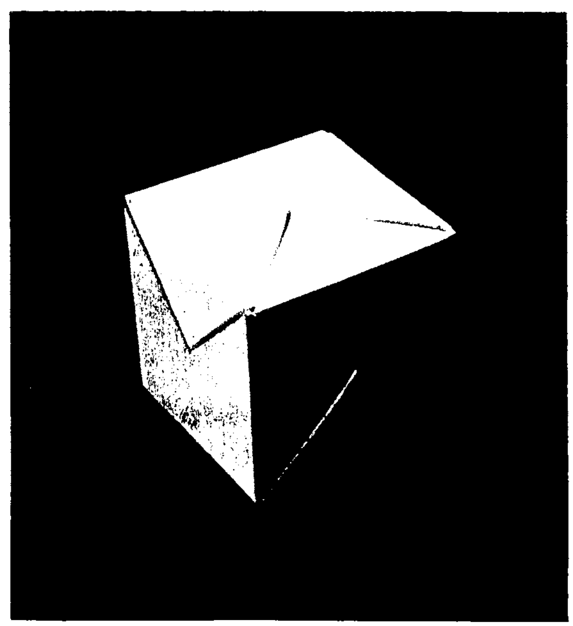


12. metà asse interno

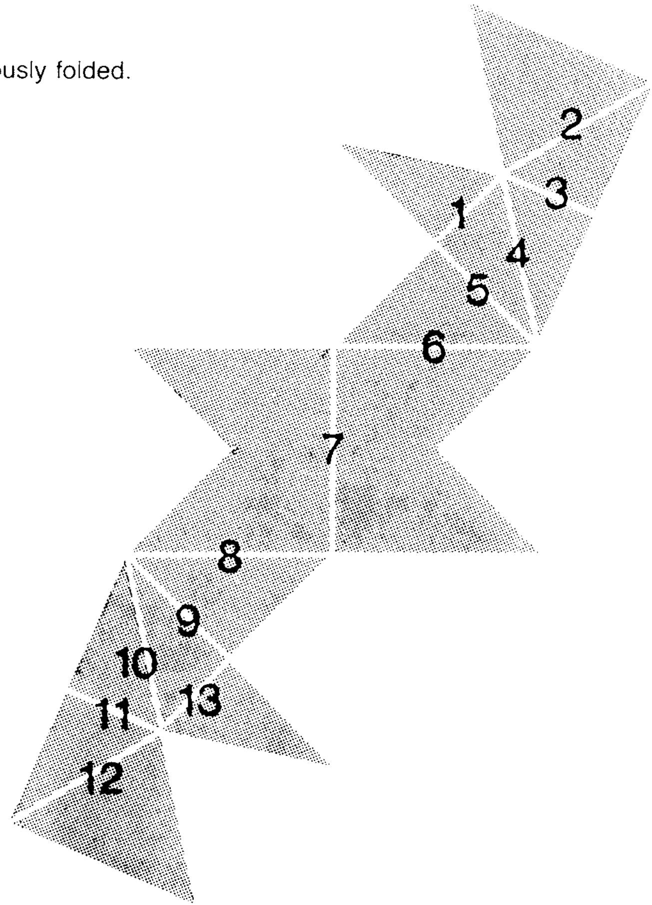


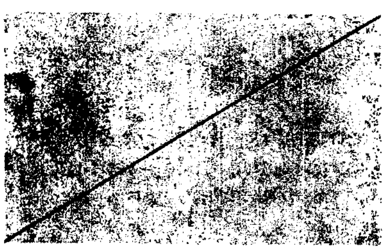


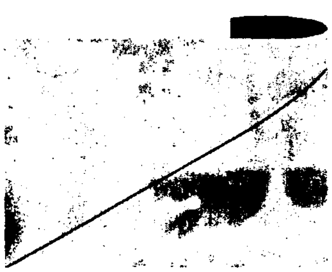
13. edge
The rotations are of 90°.

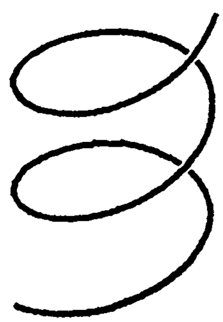


The same fold-out can be variously folded.

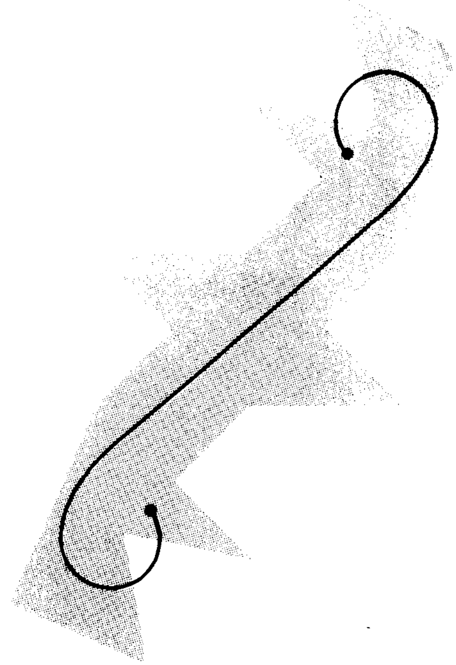


A


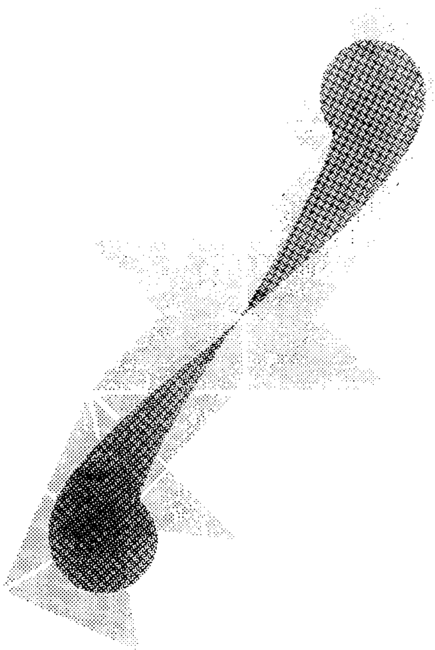
B


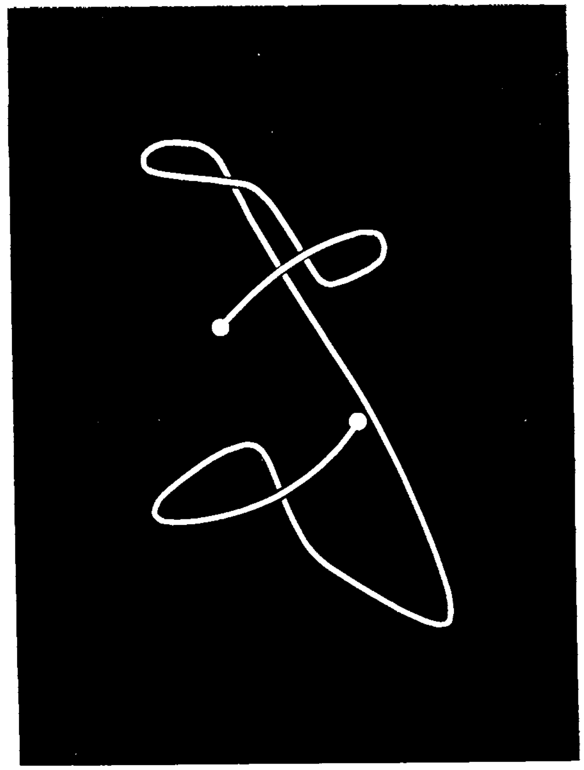


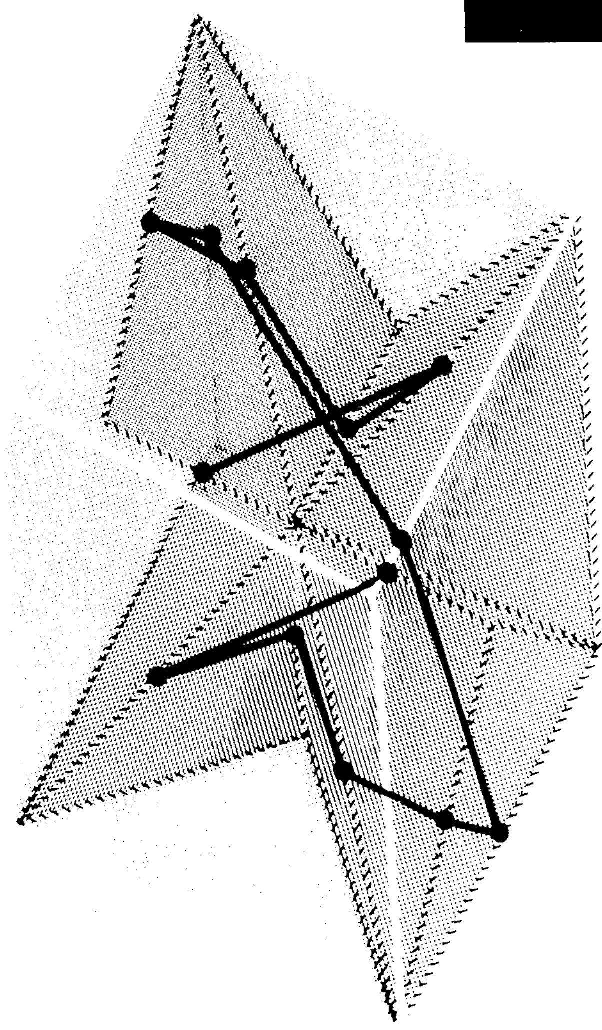
What is a helix? it's the spontaneous curve drawn in the air by the seeds of the linden tree when they fall off the plant;
or by the earth, that, rotating in space around the Sun, is moving on the advancing axis of the Sun towards the star Vega;
for the painter Paul Klee, it is "...the most pure form of movement we can think of".
In geometry, in its simplest form, is the cylindrical unfolding of the diagonal of a rectangle.

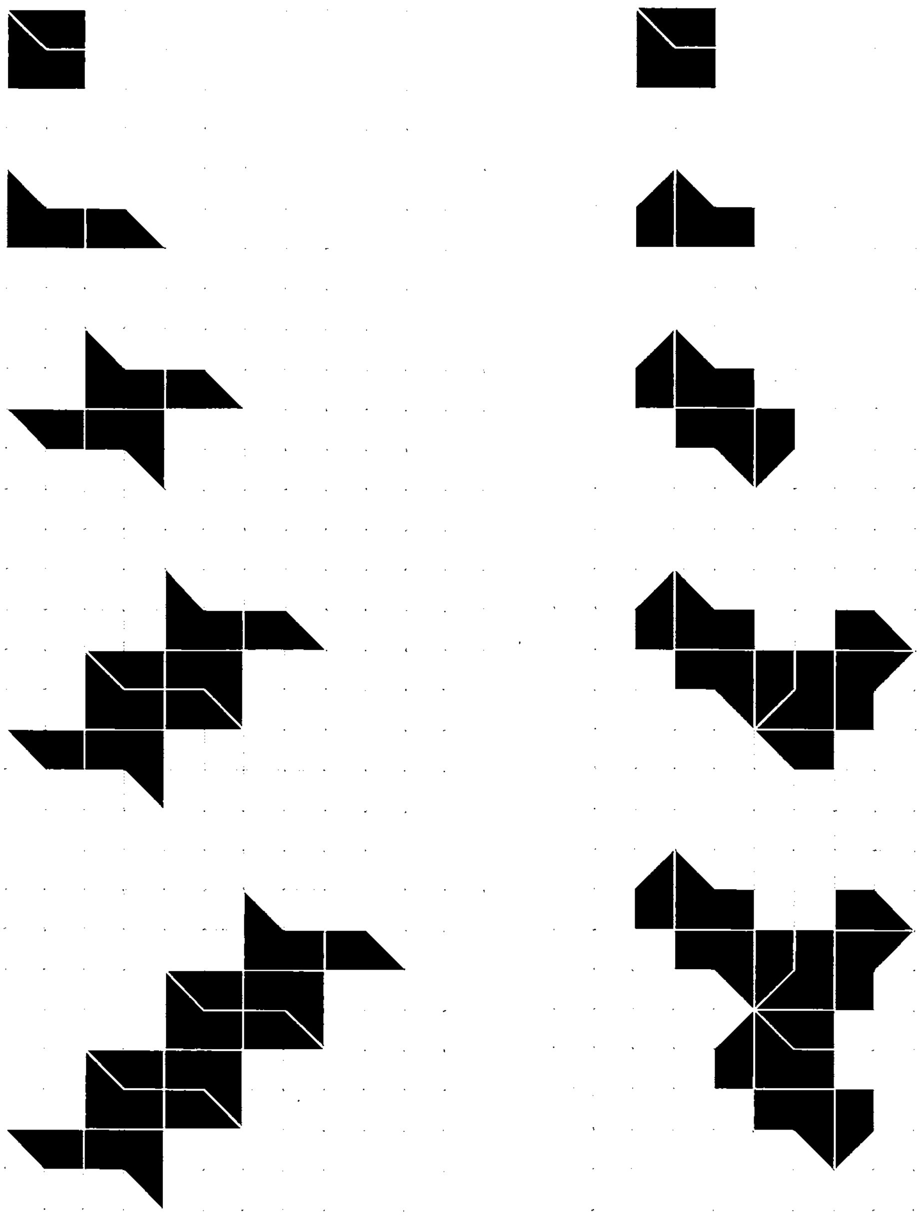


Let's consider the planes as articulated in two groupings (1-7 and 7-13, see previous pages) about the center of rotation 7; their overall shape is similar to an *s* which becomes more visible if we connect the points from 1 to 13 with a curv, or by drawing two regions that delimit the dotted tracing, thus accentuating the two-arm cyclic course of the folding.
The composition of the folding movements with which we transport the planes into space, rotating them about the axes 1-13, results in a trajectory which is shaped like a helix.
With this spatial map that unifies the sequence of the movements, we define the folding in three-dimensional space.
The representation of maps of the folding movements has as a goal the control of the entire process, towards its future semplification.

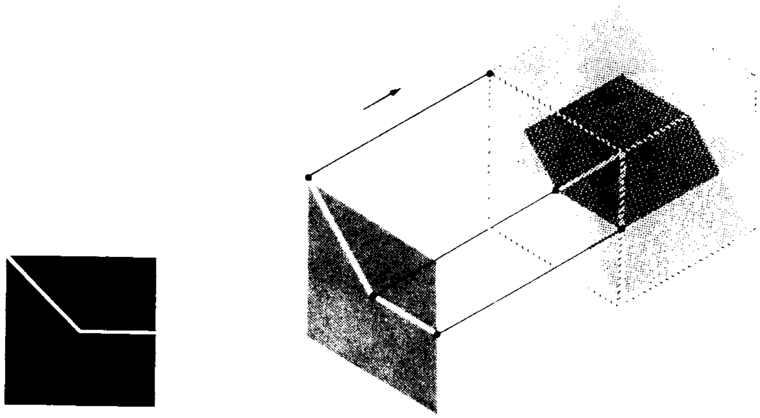




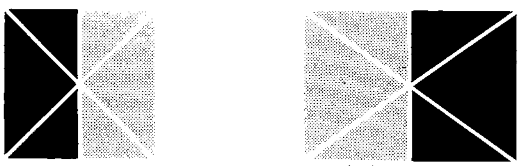




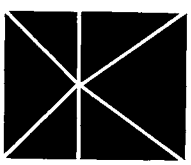
Other examples: the square is sectioned in two parts of different areas and follows the half-diagonal, half axis path. A series of fold-outs (about ten) results from the combinations by rotation of these two units. We can see two of the examples in the figure shown above.



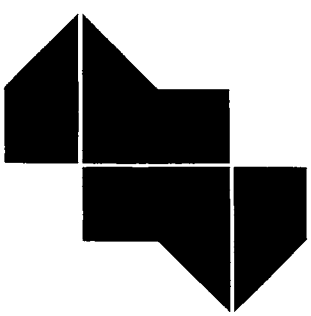
Each section of the square, as well as any path that we can follow on the reticulum, has dimensional correspondences with the internal planes of the cube;



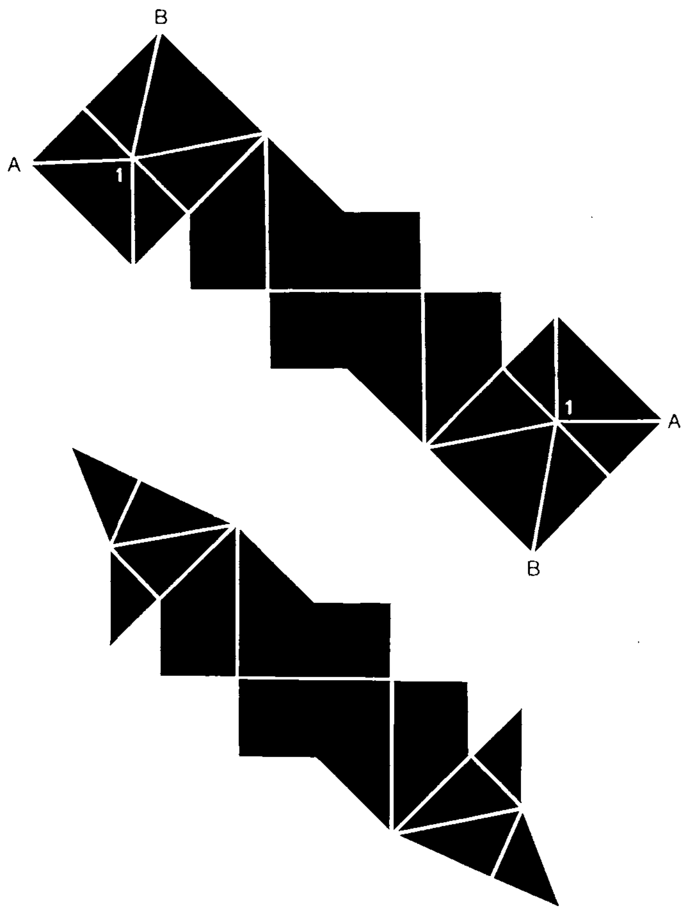
in this case square and rectangular semi-planes will be combined together.



The resulting figure contains four semi-diagonals which connect at the middle point of the axis now common to the two semi-planes;



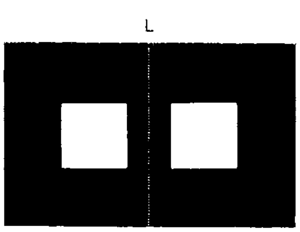
we combine now two of these rectangles to the external fold-out,

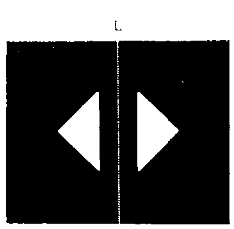


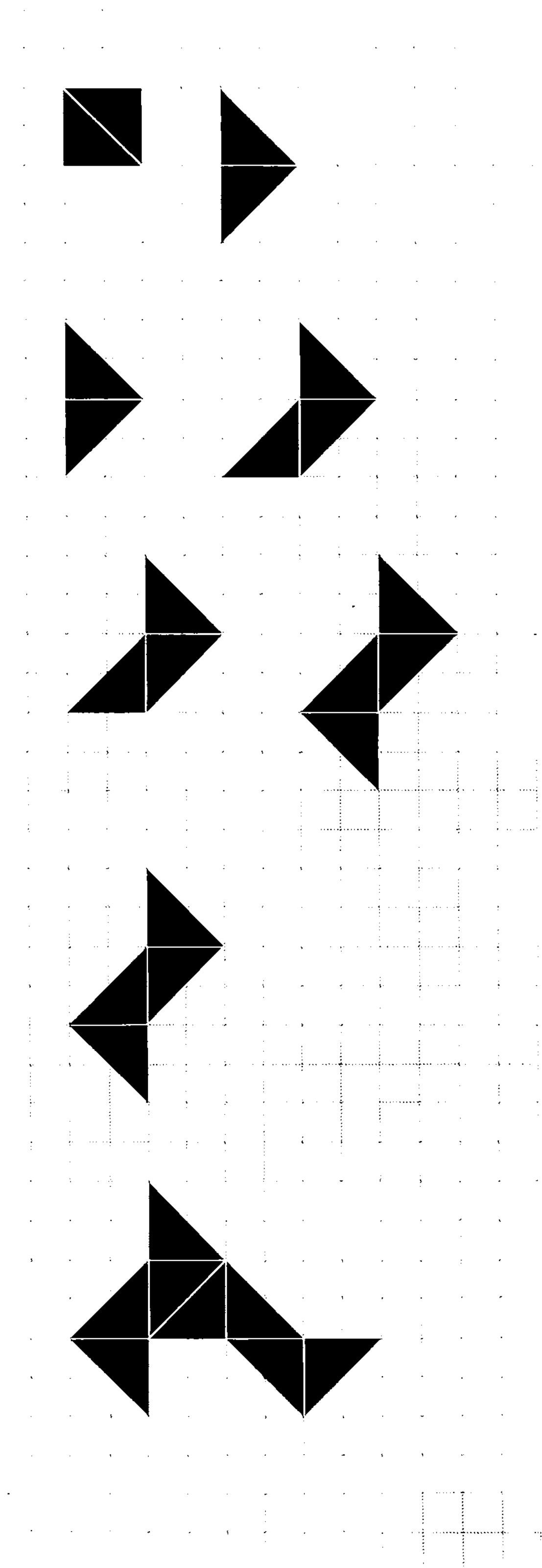
and, with rotations of 70° of the A–B–1 triangles, center in 1, we have now defined the cyclic configuration..

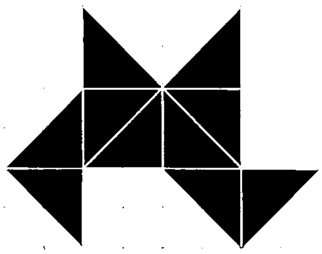
THE PAIR OF MODULES

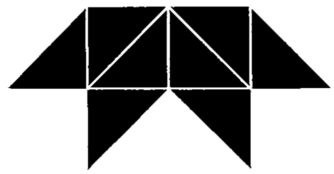
To discompose a figure in two parts is also a way to originate a pair. All the fold-out configurations realized until now are nothing else but a continuosly varied combination of pairs.
Per se, each unity has its own type of symmetry and when we ask ourselves with respect to what a figure is symmetric, the known references are point, line and plane. If we examine the previous sections of the square we see that the components of the pair are not symmetrical with respect to one another, but we can give a specular comrade to each of them through a reflection operation. Not all the modules though, are specularly symmetrical, that is identical to their own image reflected in a mirror. Many of them bear the same relationship that exists between right hand and left hand. One is the specular image of the other but neither can be overlapped onto the other.
In the facing page is a square which is sectioned in two parts along the diagonal. The two rectangular isosceles triangles forming the pair are one the specular image of the other but each one cannot be overlapped (each one of the two does not entirely occupy the geometric space of the other). By rotating only one of the two with the analogous process previously used, we obtain again three hexahedral fold-outs; (   ) if we instead double both triangles with rotations of 180° around centers 1 and 2, we form two new pairs, each of them being one sixth of the external surface of the cube. With two successive rotations of 90° of the two pairs joined together we determine the six-piece configuration. Every point of the square to the left of L, has its symmetrical correspondent in the square to the right. This happens when a reflection operation is executed. The two squares can be overlapped. Though, in this case the figures are specular but become asymmetrical if we overlap them. We can say that we have a left triangle and a right triangle that are enantiomorphic.

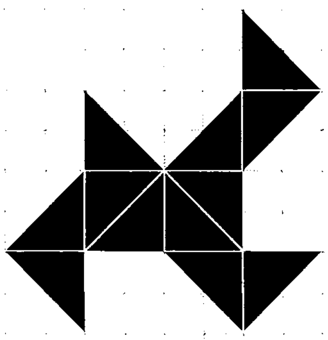


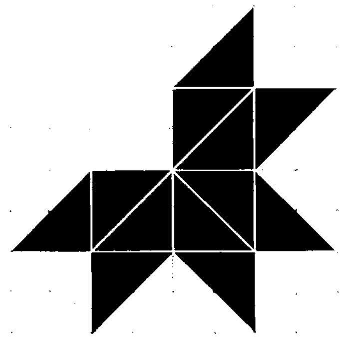


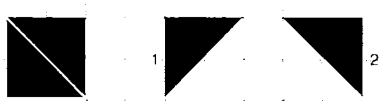


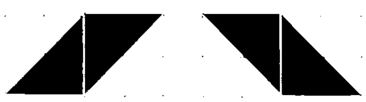


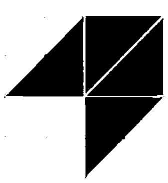


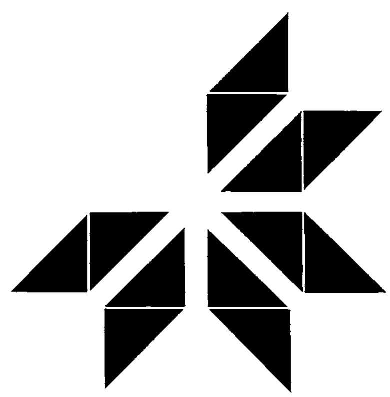


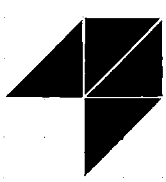

1
2

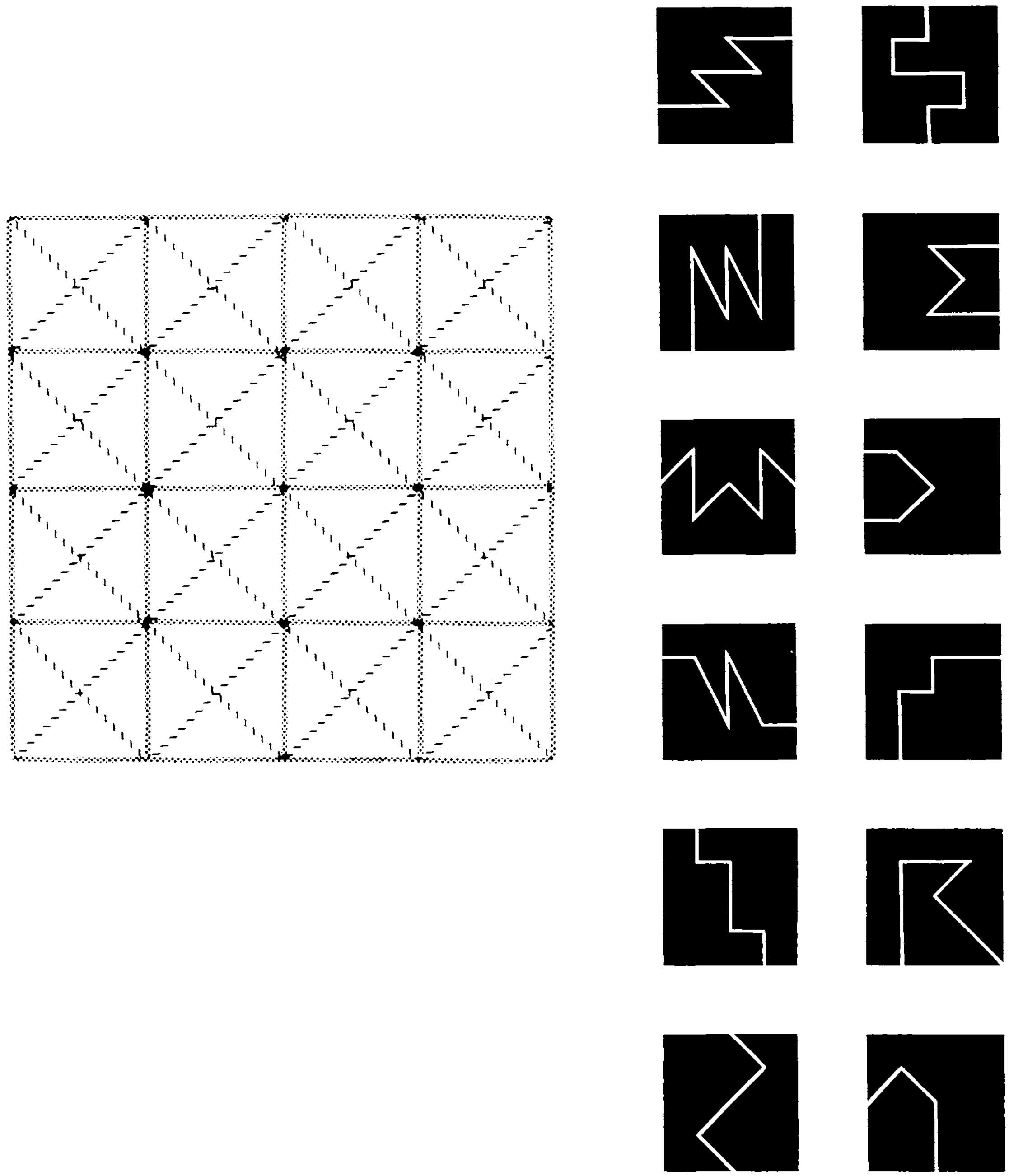




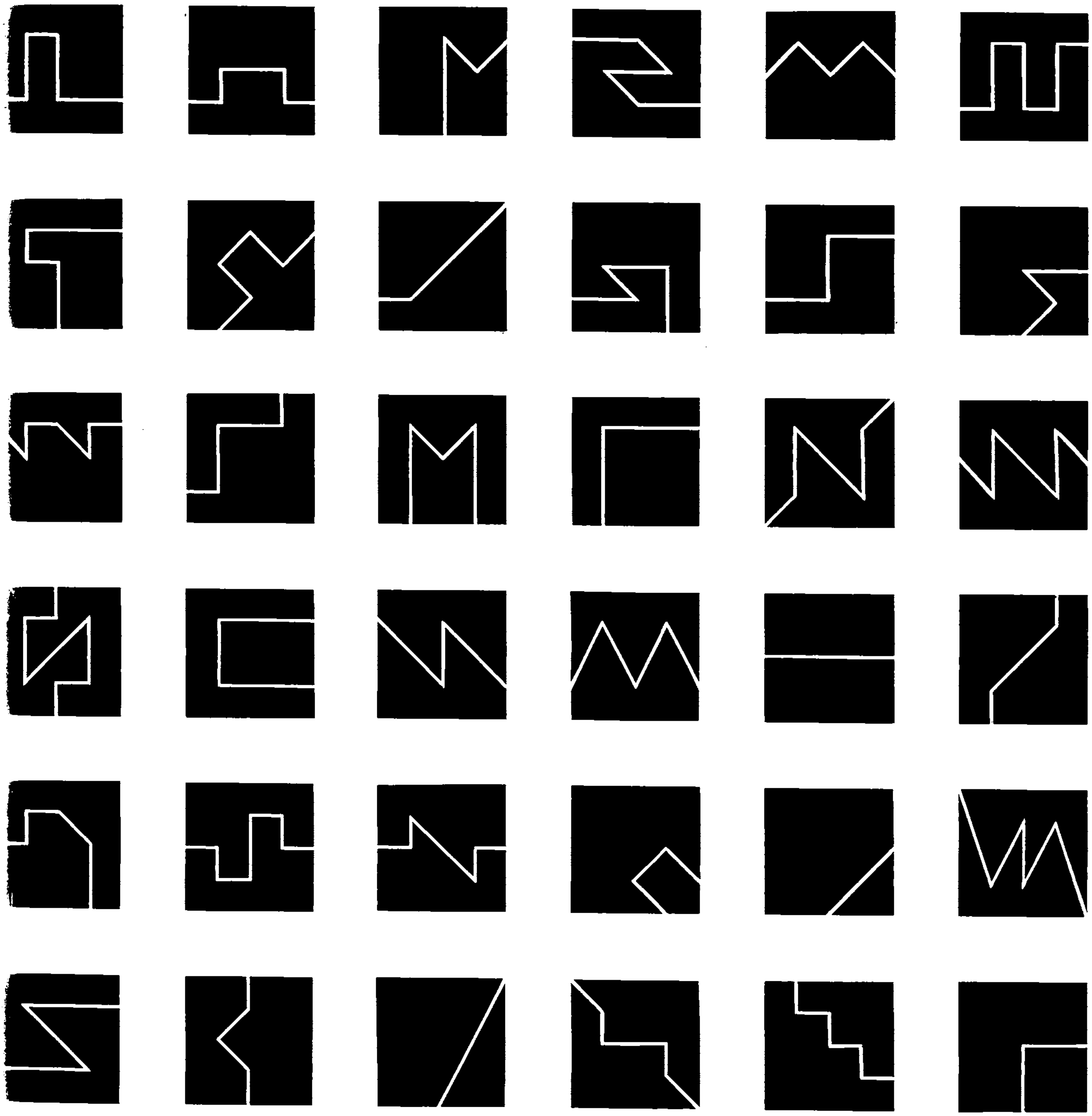




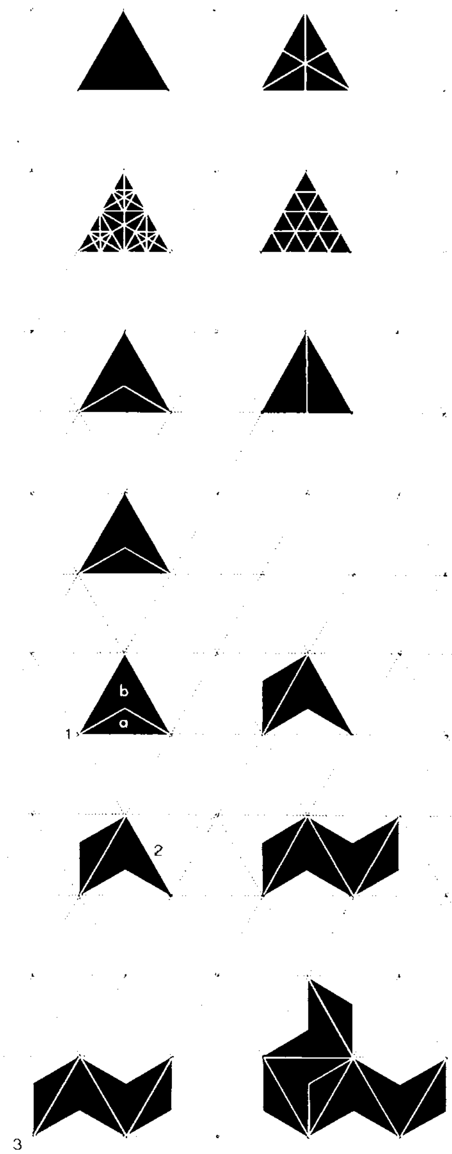




A few examples of division of the square in two parts. They are obtained by following the reticulum along orthogonal and diagonal paths. Also mixed paths of orthogonal and diagonals with respect to the sides of the total figure were used.



SEQUENCES OF ROTATORY SYMMETRY DERIVED FROM TRIANGULAR RETICULA



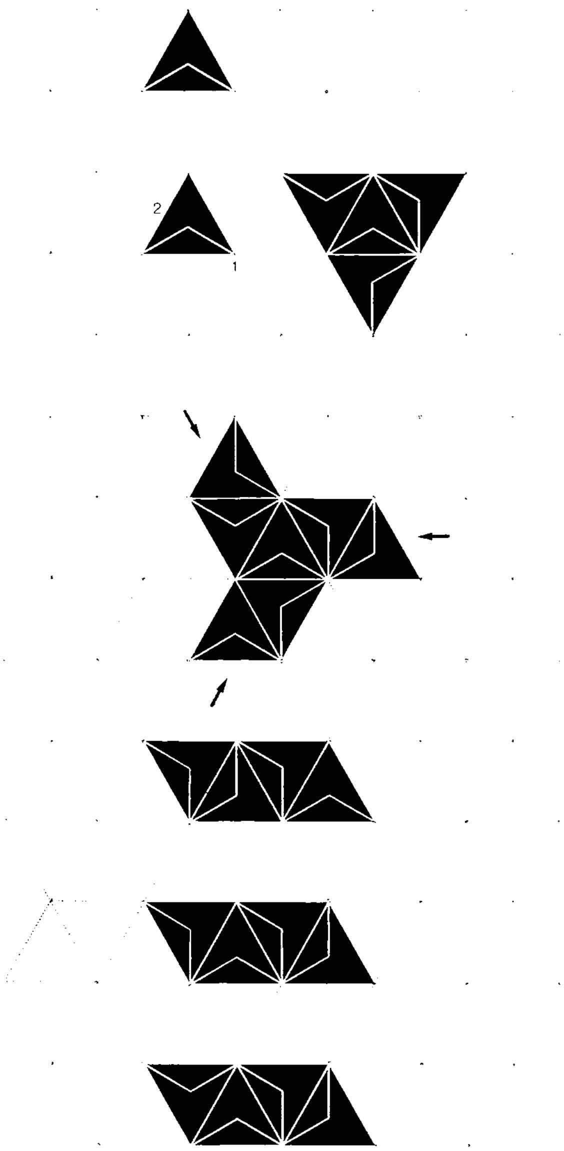
1. Trace the three median lines on the equilateral triangle by passing in the center (30°-60°-90°); use the alternate reticula of median lines and straight lines parallel to the sides, and reticula with a modular structure in relation to the sectioning itineraries that we want to follow;

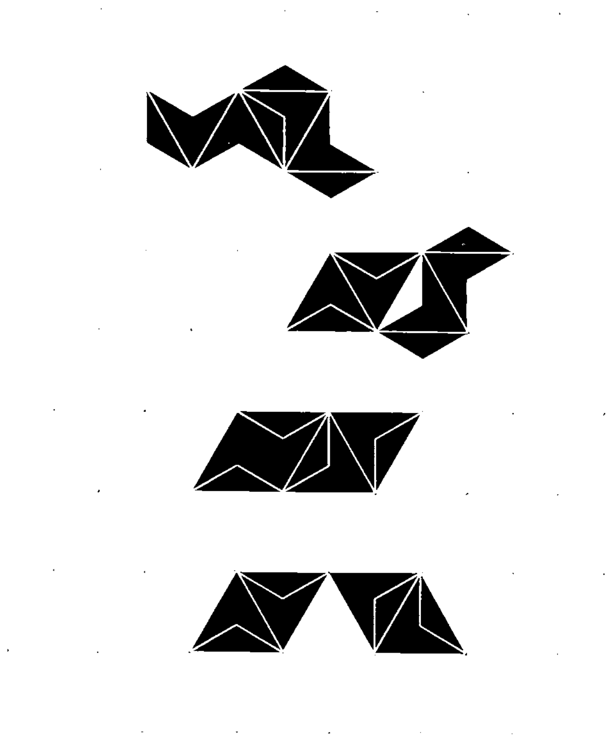
2. section the equilateral triangle in two parts:

rotate part *a* of 60° (clockwise or) counter clockwise with center in apex 1;
rotate *a-b* of 180° around center 2 (intersection of median line with side);

rotate 60° the thus obtained four-part group, once with center in 3 (common apex of *a-b*).

The tetrahedral plane surface (external fold-out) is defined, on the reticula, by 3 rotations around 3 symmetry centers; the plane fold-out groups are 2, each constituted of 2 equilateral triangles. The bisected equilateral triangle, having specular symmetry, is transformed, because of rotation, into a variety of configurations having cyclic symmetry.

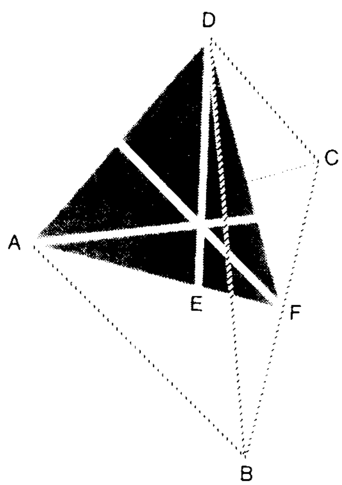


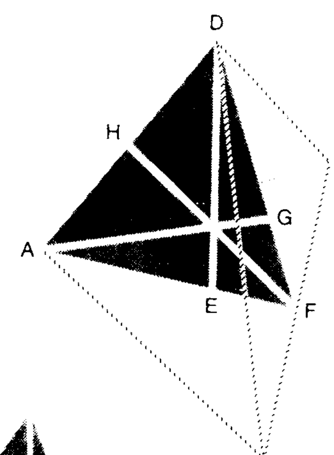


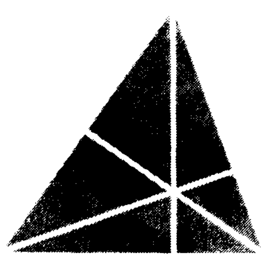
We can also use the following scheme: the triangle *ab* is rotated 60° clockwise and counter clockwise around 1, and 120° with center in 2; with three 60° rotations of the three external triangles joined to the central triangle, and with center in its apexes, we obtain a triskelion-like form. Three fold-out groups are obtained from this form if we follow the three branches along three directricies of the reticulum. Each of these groups is constituted by four equilateral triangles.
By operating on the verticies of these three forms with rotations which are multiple of 360° (60°-120°-180°-240°-300°) we can find the formal conbination of two groups of tetrahedral fold-outs; these can be discomposed and recomposed in various numerous configurations of which we can here see a few examples.

INTERNAL PLANES OF THE TETRAHEDRAL SPACE

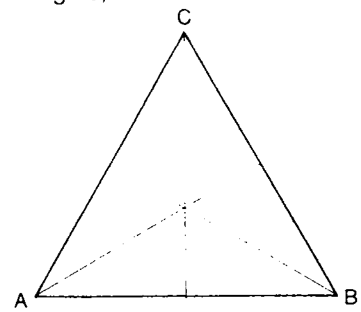
The tetrahedron has four verticies, six edges, four faces and no diagonals. In the regular tetrahedron ABCD the height D–E falls on the center of gravity of the equilateral triangle ABC. The straight line AF passing on E is a median. The isosceles triangle AFD contains two directricies of the internal space of the tetrahedron which are constituted by twice the height (DE,AG) and by a median that unites two opposite edges (FH). The triangular planes are six (isosceles triangle) of which the heights and medians intersect in the center of the tetrahedron (orthocenter of the planes).



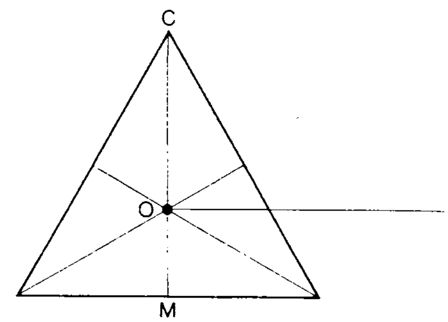




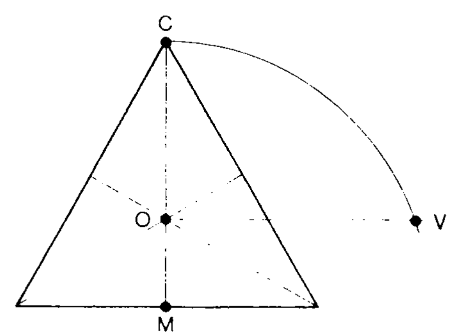
Contruct the equilateral triangle ABC and trace the median heights,

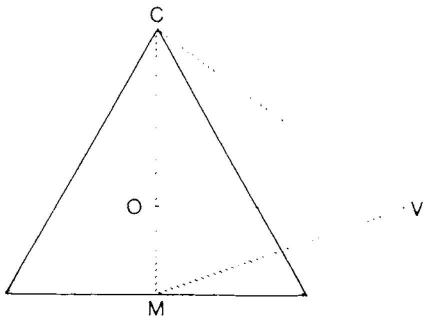


trace a perpendicular to the height MC, its origin being O,

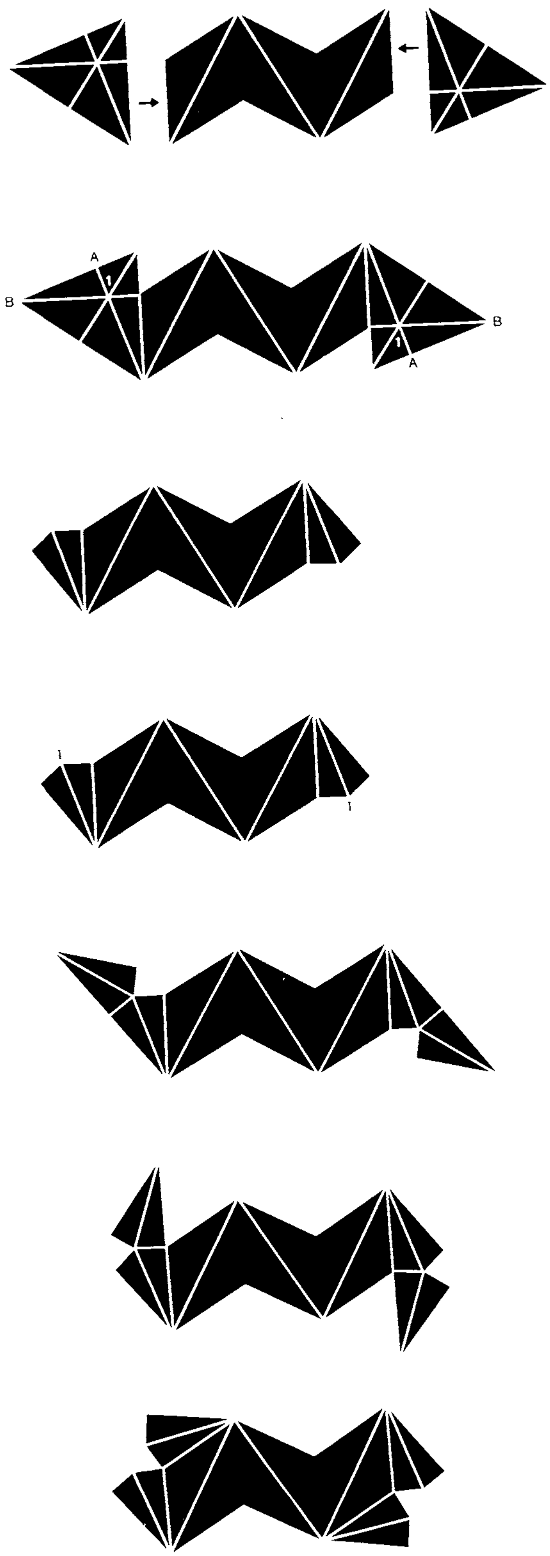


with center in M, median point of the side, and with the compass aperture MC, trace the arch CV; the points MCV are the verticies of the isosceles triangle, and the segment VO is the height of the tetrahedron.





The line that connects point V with the center O of the equilateral triangle, is the height of the tetrahedron contructed on the same triangle.



These planes are variously connectable to the groups of external surface in conbinations that tend to accentuate the cyclic-type formal aspect of the planes themselves:

1. unite two isosceles triangles containing the internal spacial directricies of the tetrahedron to one of the two external surface fold-outs, in such a way that there be a dimensional correspondence between the parts that we want to connect;

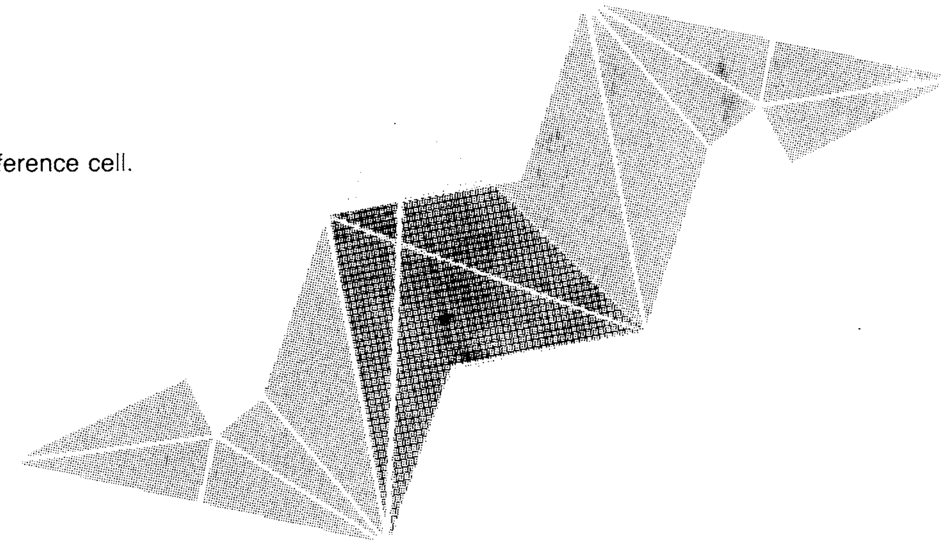
2. rotate two rectangular triangles 1–A–B 110° counter clockwise around center 1;

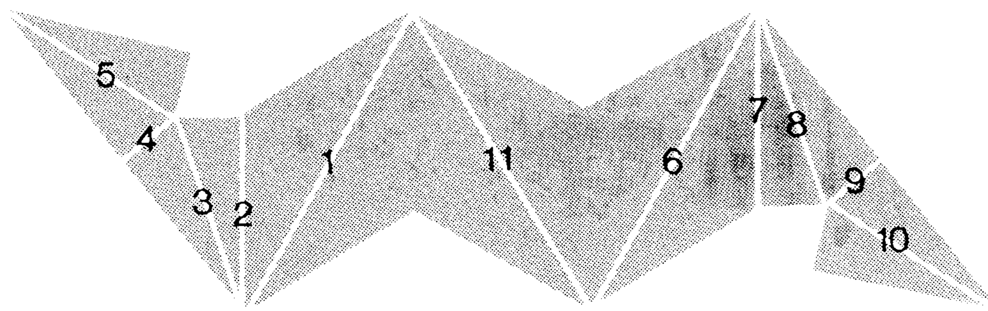
3. rotate the such obtained pairs 145° around the same centers. Thus we determin a plane group of total surface, which corresponds to half a tetrahedron.

ROTATION–FOLDINGS OF THE TETRAHEDRAL FOLD–OUTS

Preordered folding sequences of the new total-surface fold-out follow the composition of the internal and external planes of the tetrahedral body.

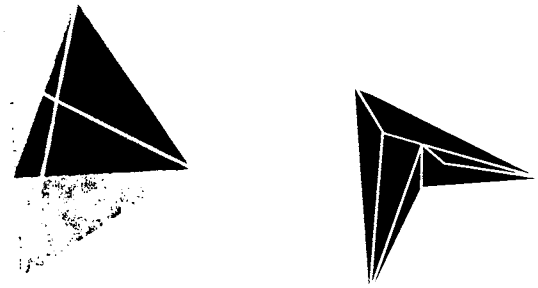
Tetrahedral reference cell.

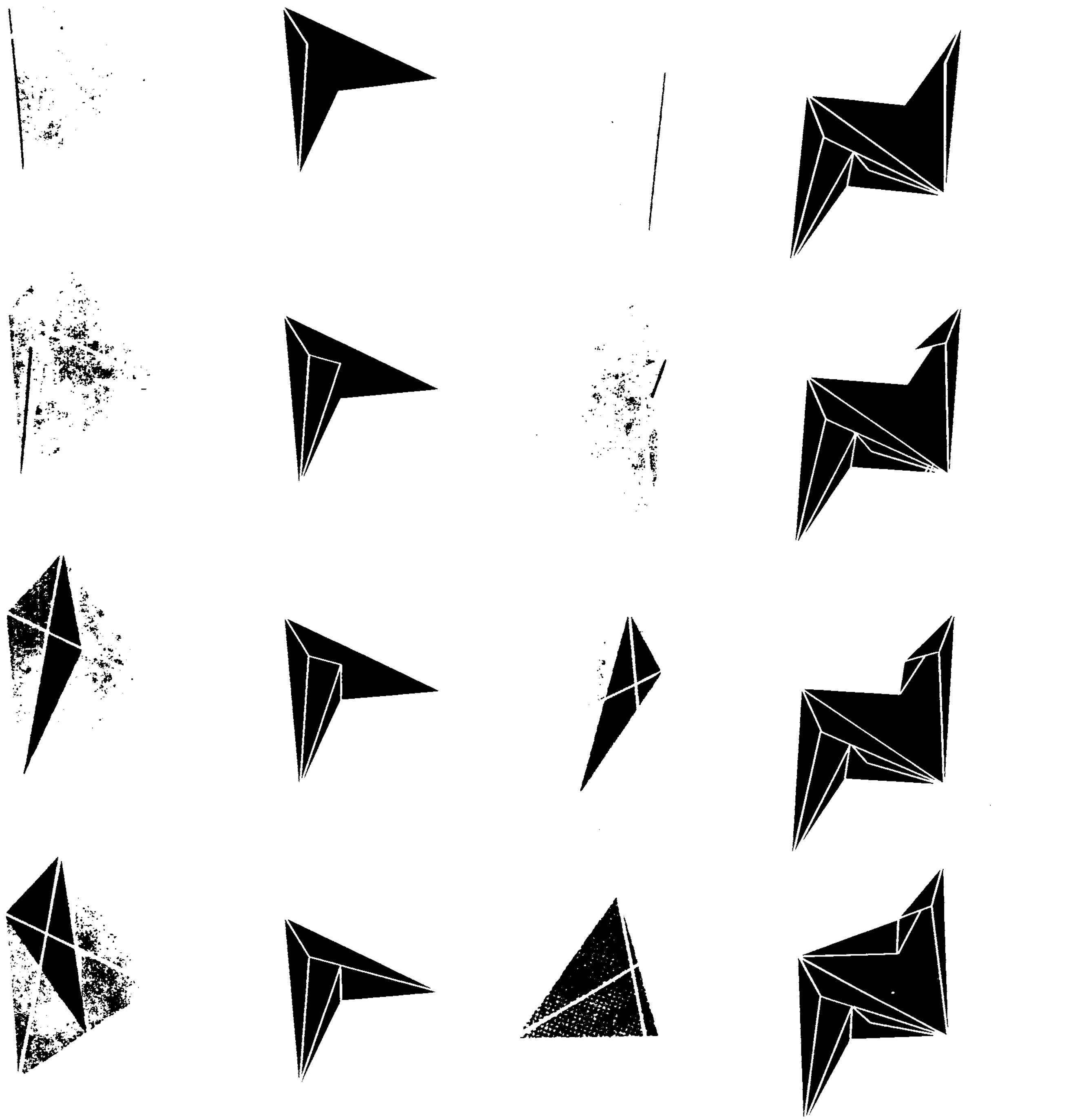


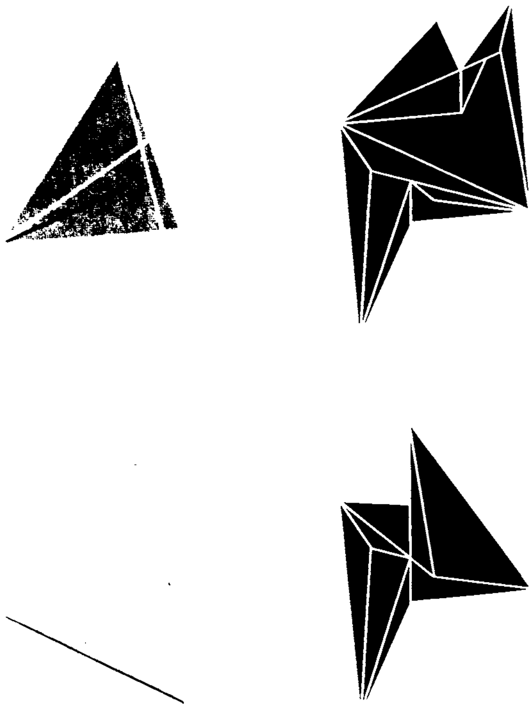


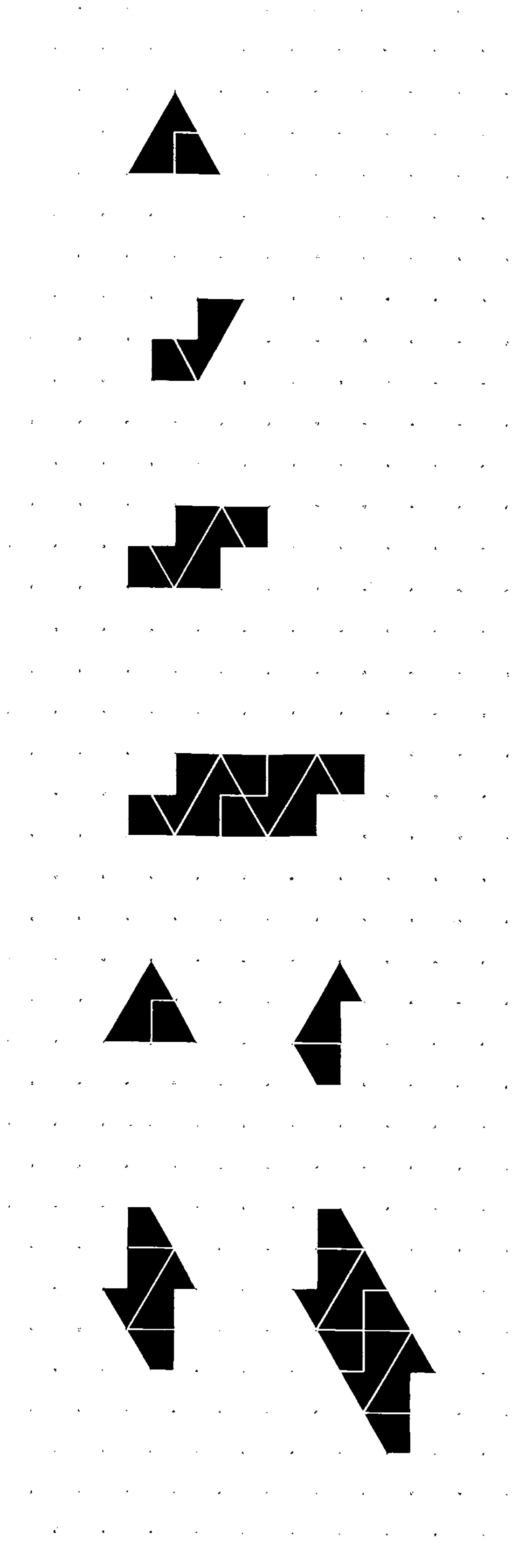
By folding half of the fold-out from 1 to 5 we describe a helicoidal curve in space; through a rotatory movement of the series 6-10 we originate a second curve similar to the previous one; with the 11th fold, by connecting the ends we close up the fold-out.

Progressive addition of the plane sectors, each one referring to its rotation axis; the angles of rotation are in the order of 120°, 90°, 60°, 60°, 60°, 120°, 90°, 60°, 60°, 60°, 120°.





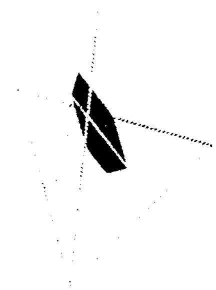




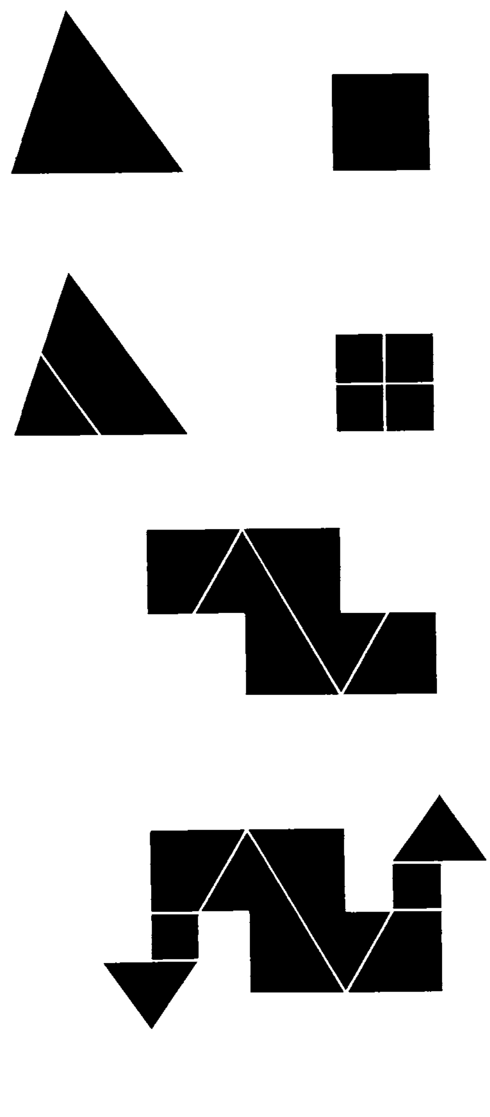
Other examples of formal articulation according to the external-internal relationship.

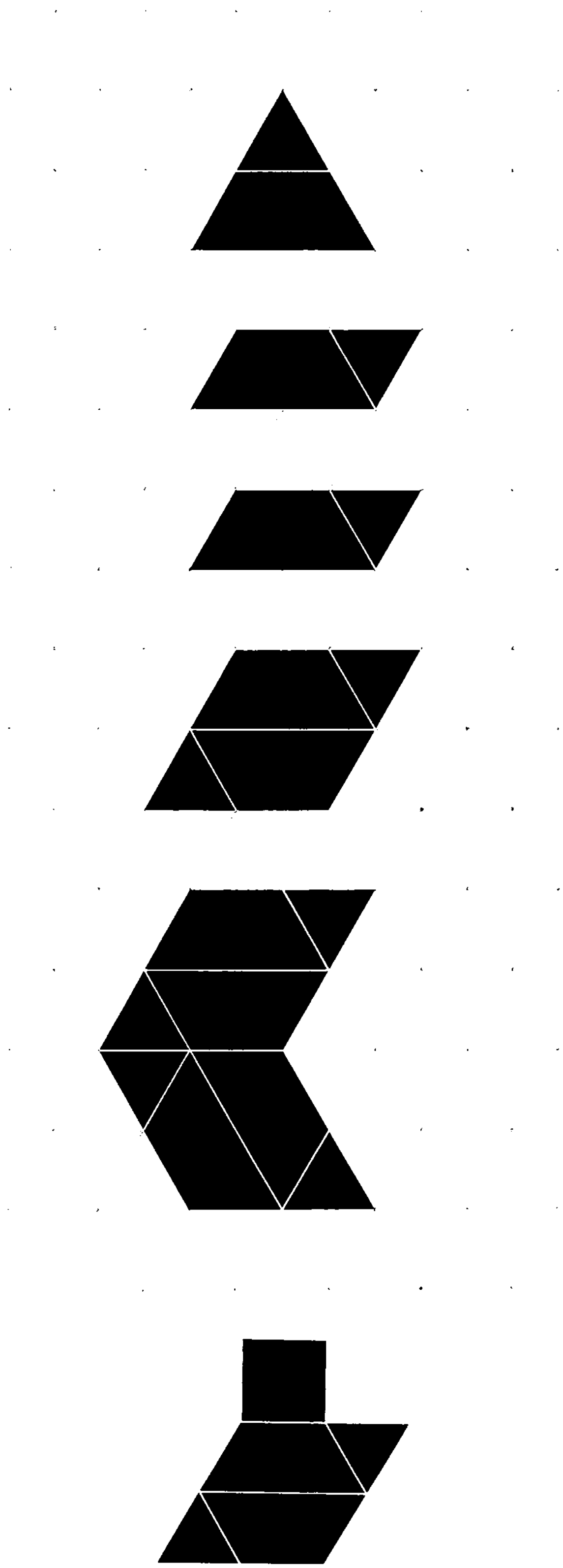
By connecting together the four median points on the four edges of the tetrahedron, we obtain a square, structural internal plane that divides the polyhedron in two equal volumes.

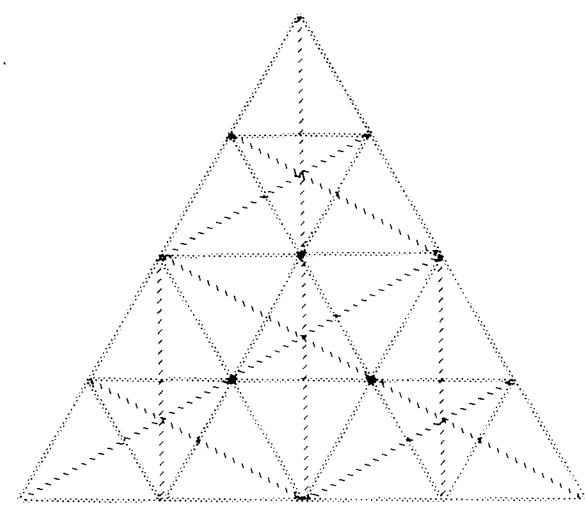
Various intersections can occur with the other triangular plane that also cuts the tetrahedron in two equal parts.



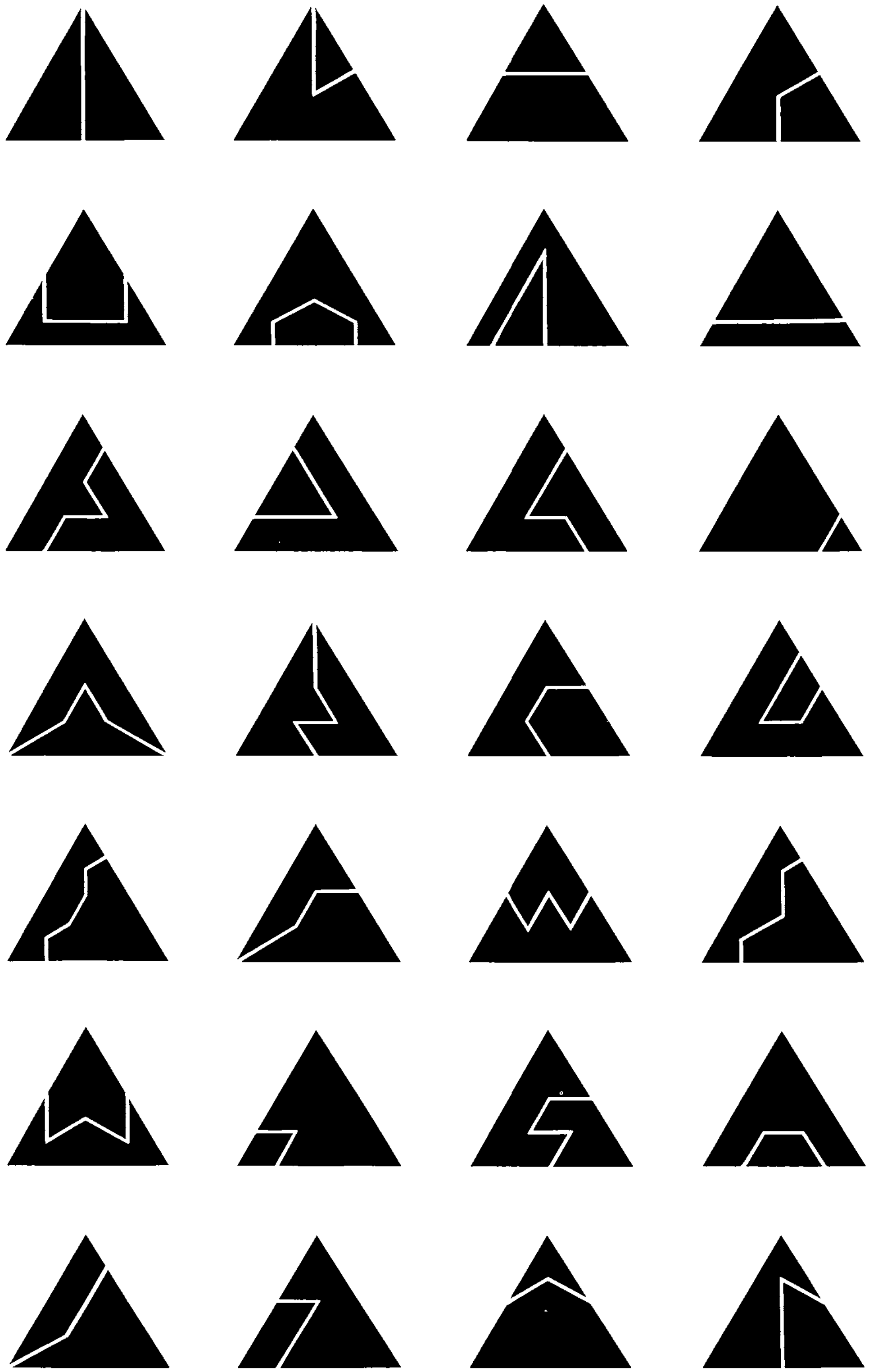
From intersections of the internal planes, which are always related to the form of the external-surface fold-outs, derives this discomposition in triangular and squares submultiples that complete the plane cyclic form of half tetrahedron.

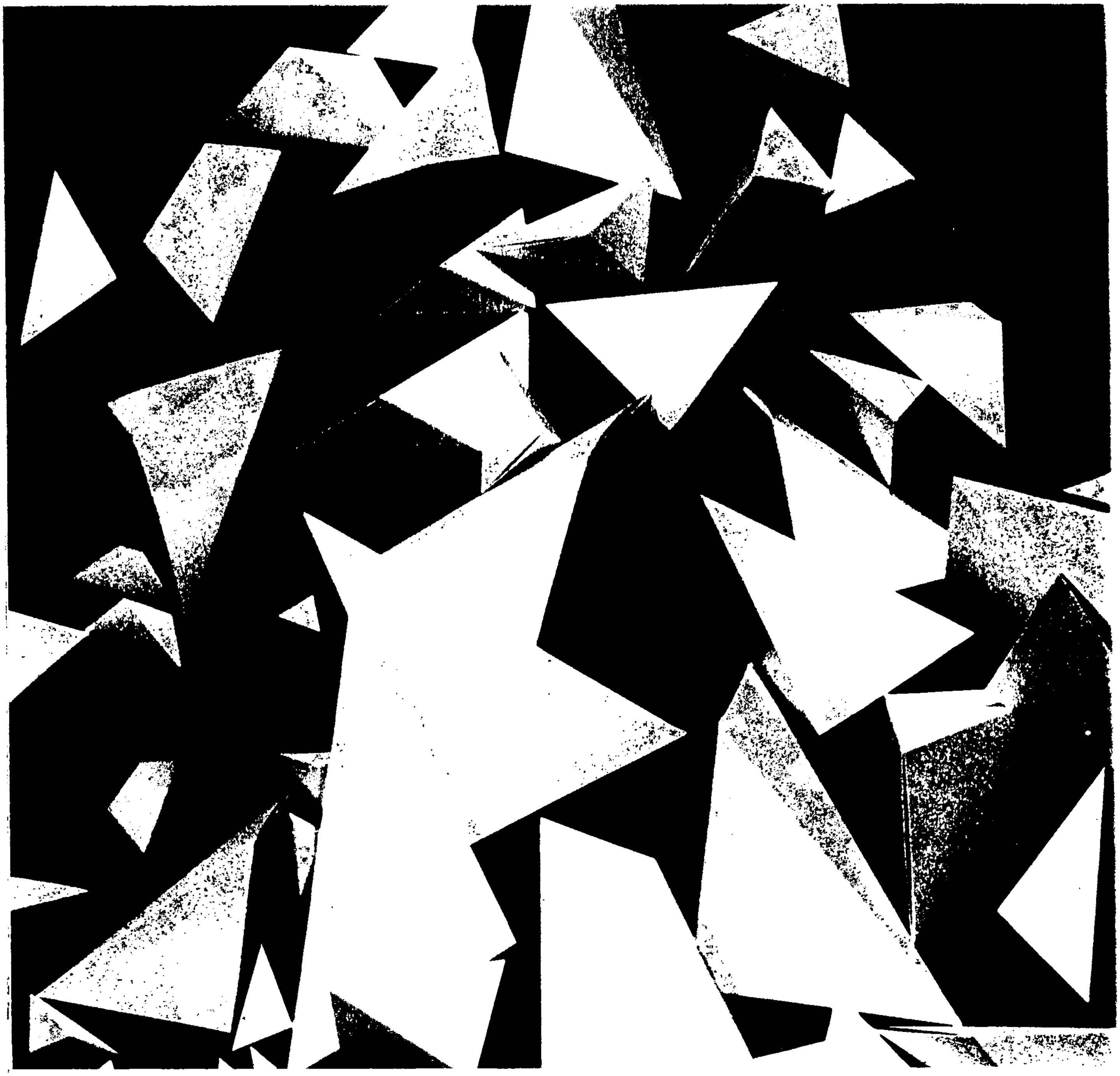




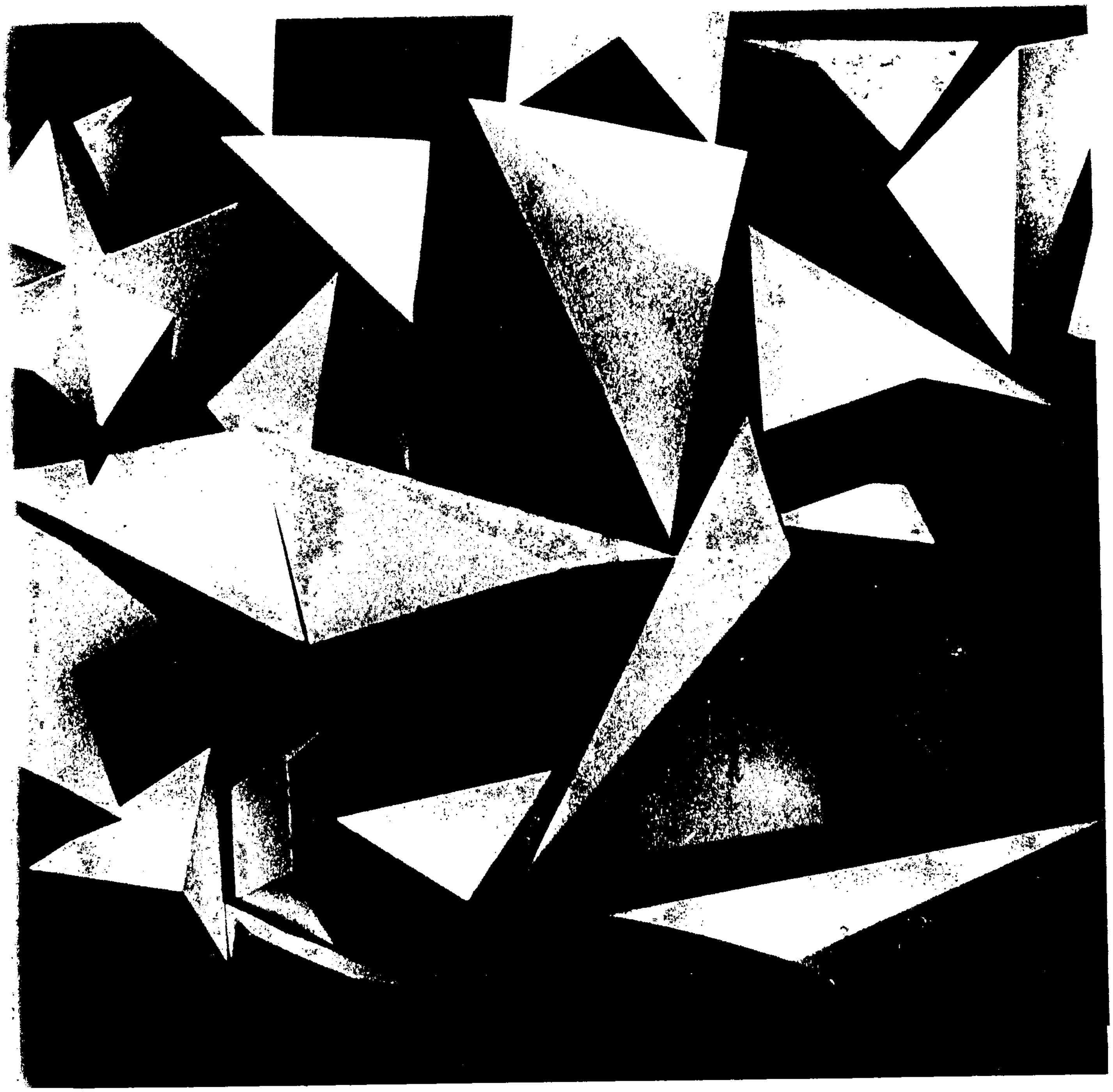


Modular reticulum of the equilateral triangle. All lines of division in two parts of the triangle shown in the facing page are traced following this reticulum. The examples can be numerous.





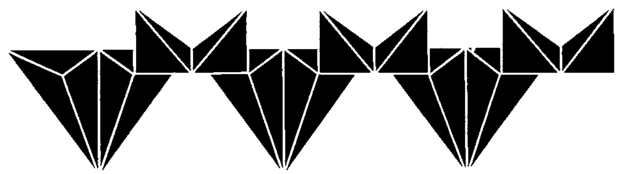
The study of the complementary relations of the sections of the equilateral triangle and of the square, the multiplication of the coupling of the forms, regulated by rotatory symmetry, bring us to the modular fold-out. In the same way that a distinct fold-out, for example a plane tetrahedral fold-out, can be spatially coordinated by referring to the tetrahedric cell, the quantitative regulation of a three-dimensional tetrahedral model will depend on the same reference. All this requires being precise about the number of modules to build, the type or 'species' to which they belong and the function of what we will make choices on.



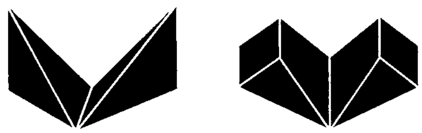
THE CHAINS OF PAIRS OF SPECULAR MODULES

1. The tetrahedron, the cube..., all polyhedra, can be considered as one of the numerous three-dimensional configurations resulting from the folding of flexible chains of modules.

2. The chains are costituted by pairs of modules in which one component is the mirror image of the other.



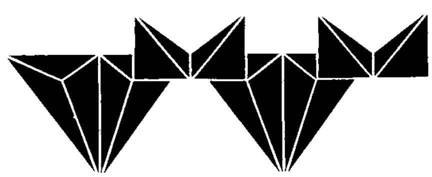
3. To build a chain it is necessary that a single unit (module) be tied by a hinge or joint to the near specular unit in such a way that it forms a pair, and that the pair thus constituted be connected to the preceding and following pairs.



4. The costruction of a pair is possible when we know the form, the number of pairs or modules, and the type of organization that we intend to applay to them.

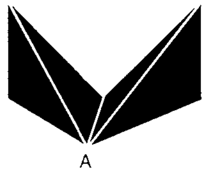
5. The line-up sequences assume a rectilinear path (or a zig-zag one); in them we can find pairs of uniformed size and similar between them (modules and submodules).

6. The chains are classifiable on the basis of rhythmic succession of the number and type of the pairs, or of the number and type of modules, of the spatial order given to the latter, and of the number and spatial position of the joints.

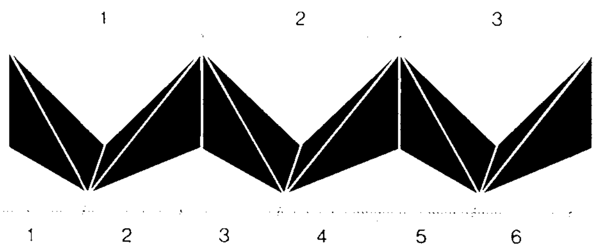


An example:

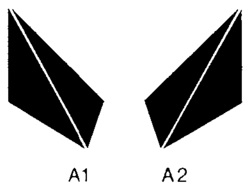
a) hexahedral modular pair "A";



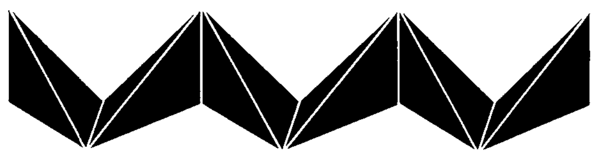
b) rhythmic sequence of pairs and modules;

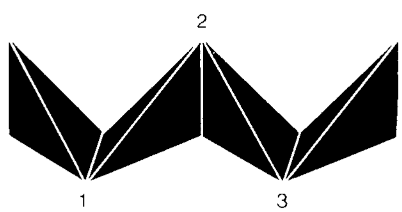


c) left specular unit "A–1" and right specular unit "A–2";

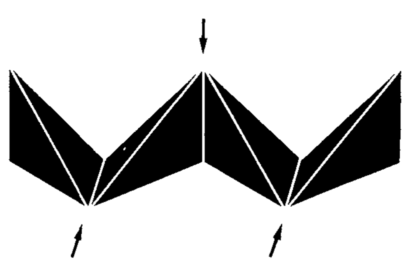


d) specular units "A–1", "A–2";



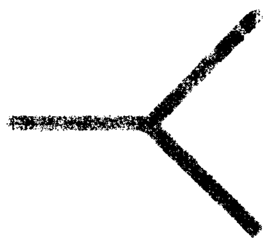


e) joints and their spatial position.

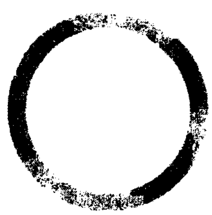


In relation to the variability of the spatial disposition of the joints and modules, we can build:

A) chains with an open boundary;
B) chains with a closed boundary;



1) the bundle chain,
2) the branched chain,
belong to A;



1) the ring chain,
belongs to B.

These are the three simplest and fundamental figures of articulated combinations of modules. They represent, in a schematic way, the top view of the chains on the plane reticula, and the linear and nodal structure of the modules in the three-dimensional reticula.

THE BUNDLE CHAIN

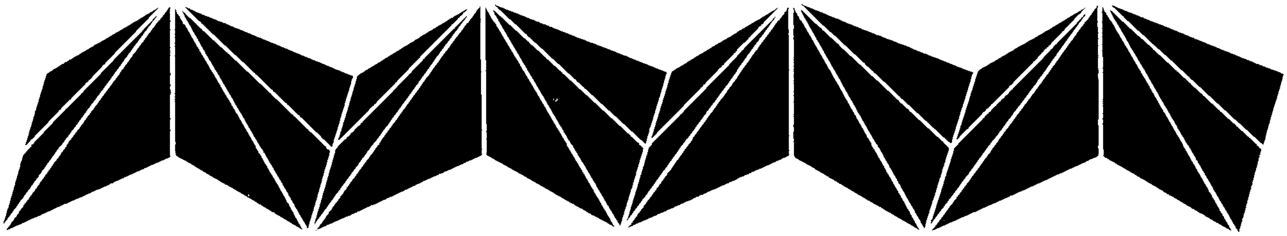
In the bundle chain the union of the pairs originates rectilinear sequences or zig-zag ones. The bundle chain must be constituted by a combination of at least two linear sequences of modules. It results from the joining of the modules through the use of flexible connections, following only one extension of lines or succession of parallel points, belonging to the planar and three-dimensional reticula. The directricies (horizontal, vertical, diagonal) along which we connect the modules following this scheme are three. The joining sides of the modules, disposed on two dimensions of the three-dimensional space, which are necessary for the contruction of a bundle chain are also three. The variety of combination, and the rotation of the modules, allow this chain to move laterally with respect to its major axis and also to inwardly fold itself. The flexibility of a bundle, branched, or ring chain depends on the form, the number of pairs, the position and regularity of space-dimensional succession of the joints. The degree of flexibility of a bundle chain is variable along its reticular structure.

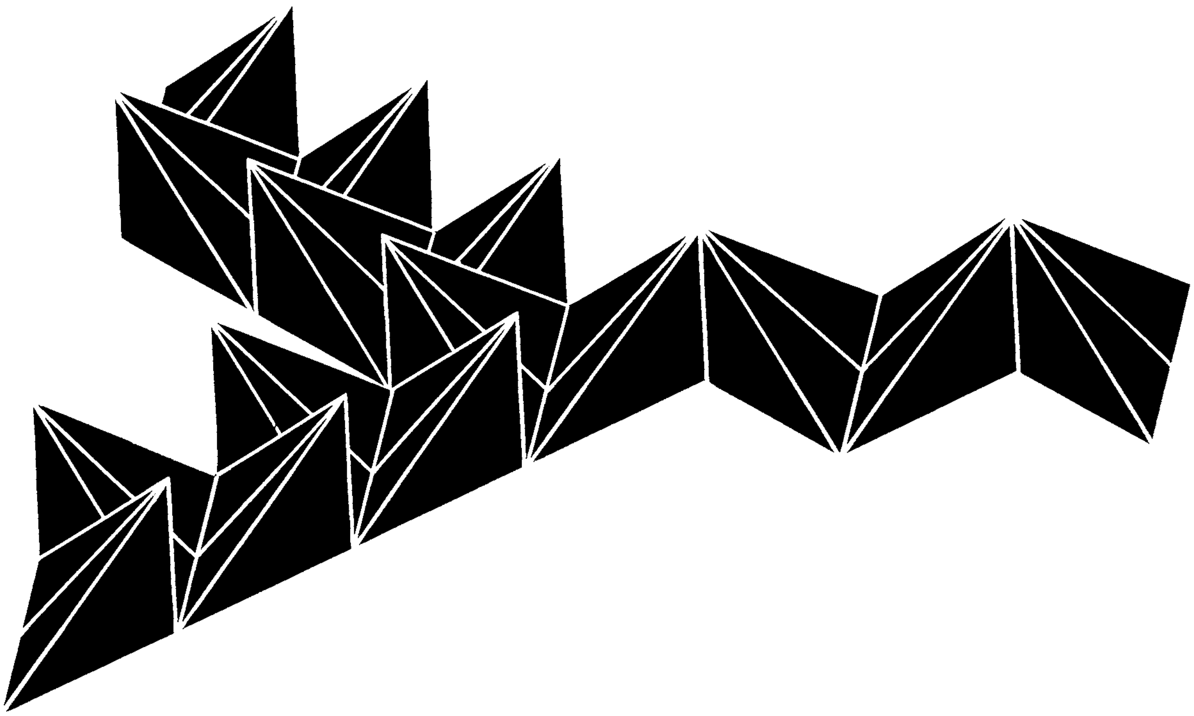
THE BRANCHED CHAIN

In the branched chain the pairs are disposed in rectilinear or in zig-zag sequences, in bundle conbinations, or mixed, compounds of rows and bundles. The linear distribution of the modules must be made following the directricies of the reticula in one of the above indicated way. The branched chain results from the addition of segments which bifurcate. The terminal modules of each segment constitute a flexible node with varied angulation with respect to the axes, upon which, beginning with a number of two, other chain segments are attached.
The joining sides of the modules necessary to contruct a branched chain are four, disposed (as for the bundle chain) in two spatial dimensions. The branched chain is variously foldable. The degree of flexibility depends on the structure of the branched segments, the way in which they are connected to each other, the variety of rotation of the modules.

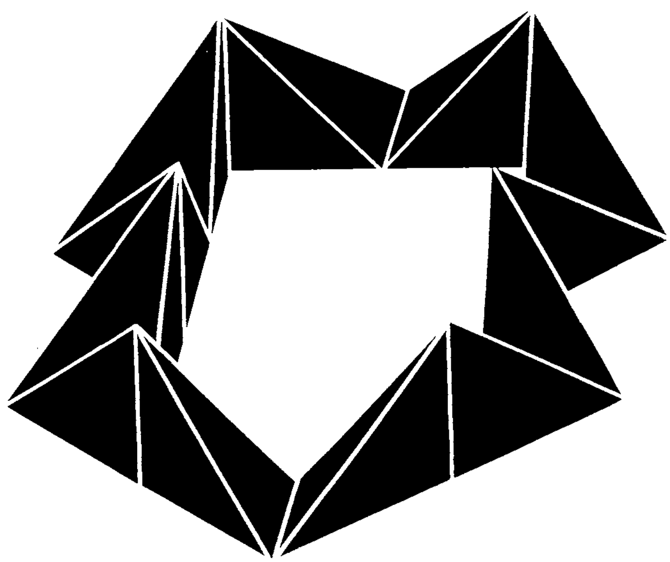
THE RING CHAIN

It is composed of pairs of modules joined in theories, linear segments, whose extremes must be united to form a 'ring'. The joining sides of the



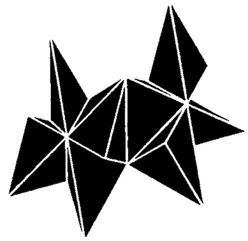


modules, indispensable to construct a ring chain are two, disposed in two dimensions. The flexibility of this chain is a function of the form and the number of pairs, hence of the regular repetition of the bi-dimensional rhythm of the joints. The combination by rotation of the modules generates a noticeable variety of geometric figures. Foldings of 360° of the whole chain are possible, determined by pairs of rotatory movements. The planar configurations on the reticula of the modules that constitute a closed chain are innumerable, always assuming polygonal forms with a tendency to the circle.
The simplest closed chain we can construct has on the plane the shape of an equilateral triangle.

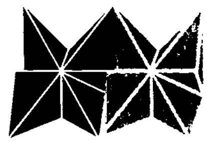


CONSTRUCTION OF A CHAIN

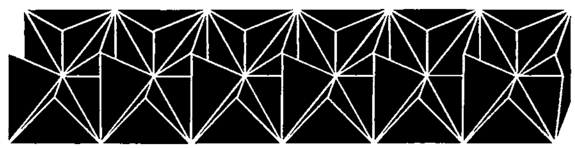
A practical way to realize a hinge consists in joining the modules together with a little piece of scotch tape.



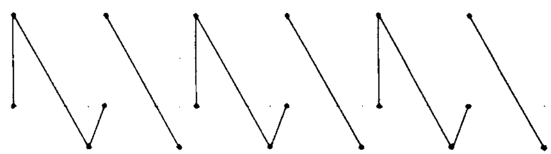
Coupling.



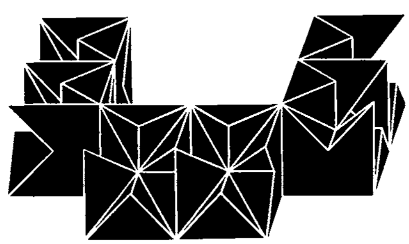
By connecting together six pairs (twelve modules) we contruct a chain.



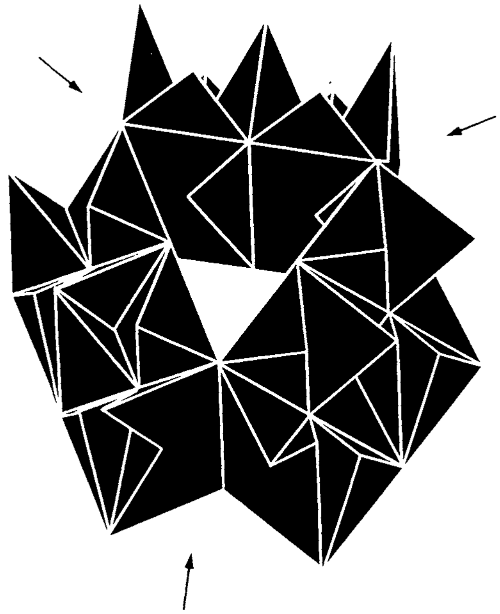
Let's now dispose the modules on a line, following the montage scheme that indicates the spatial position of the hinges.



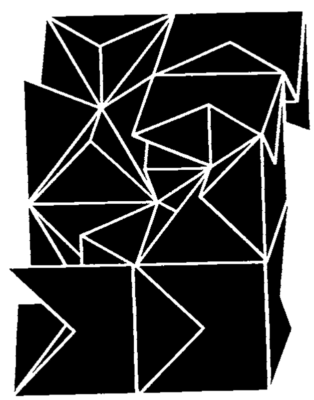
Connecting the first and last of the modules the chain will spontaneously dispose itself in the simplest obtainable form of closed chain: the closed chain with a triangular configuration.

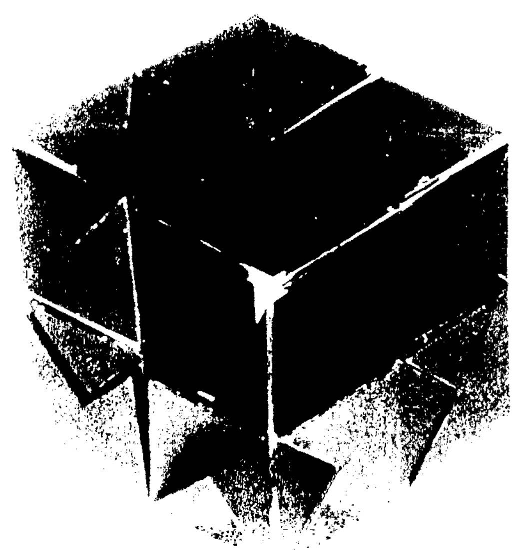


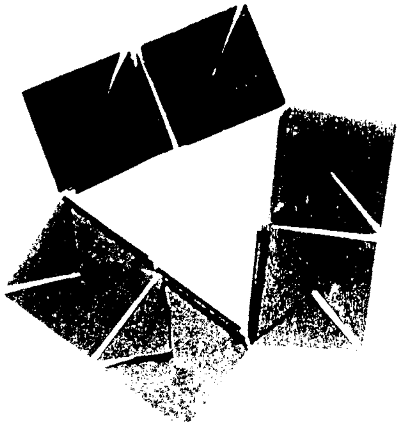
By applying, at the same time, a light pressure on the points indicated by the arrows,

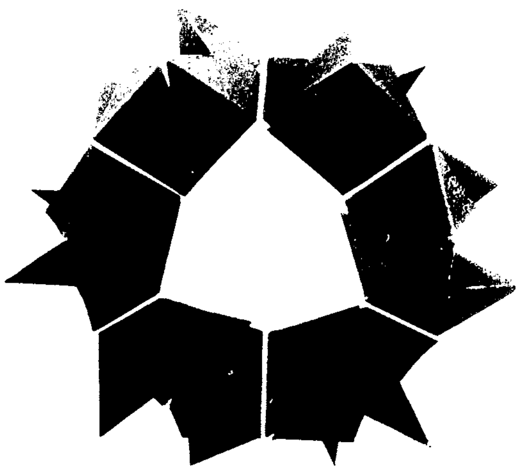


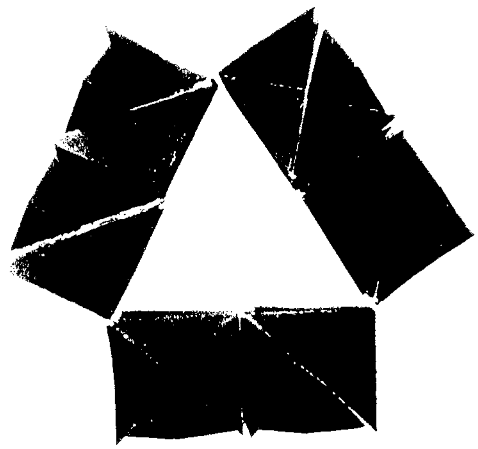
the chain will assume a cubic form.

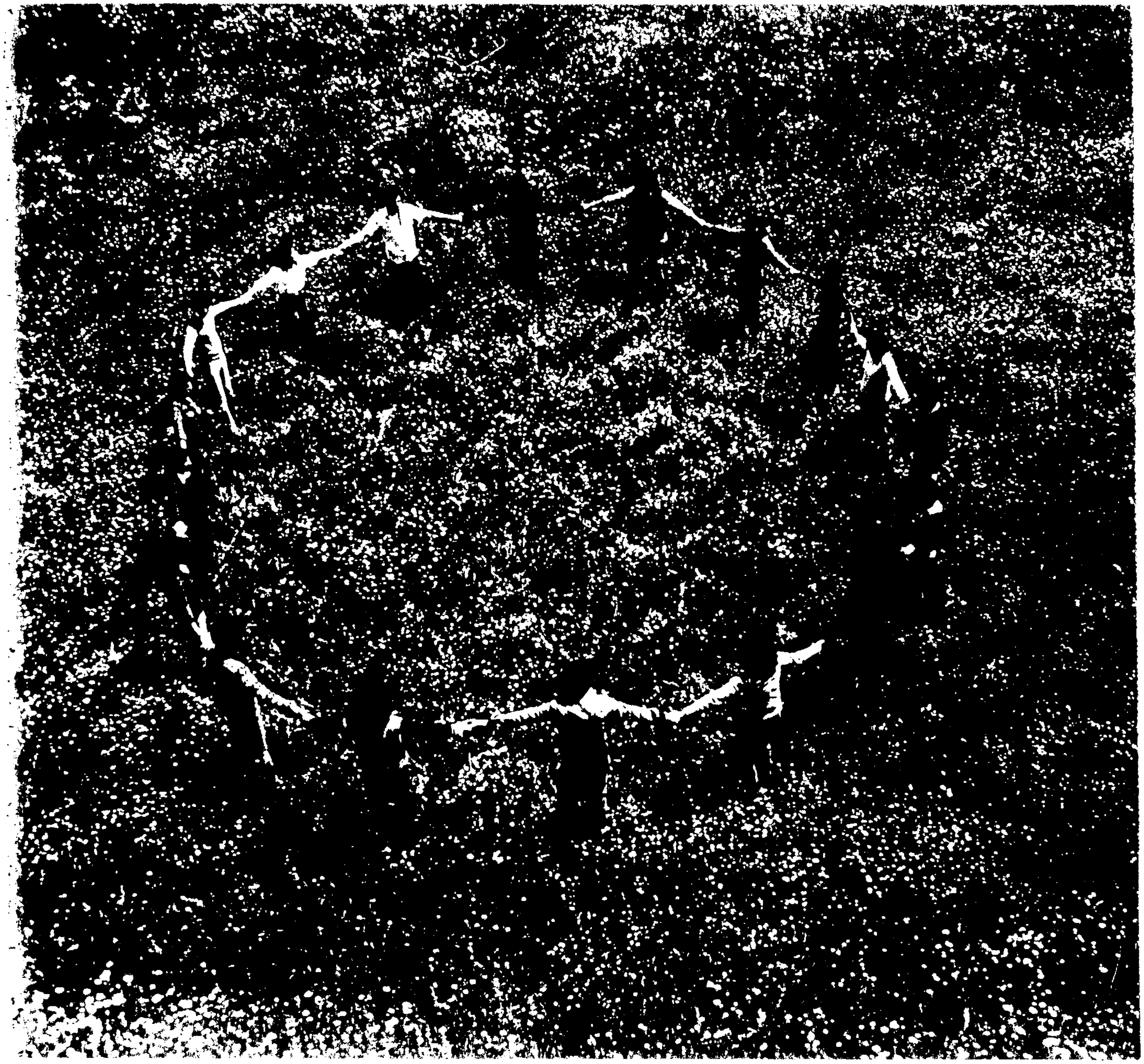




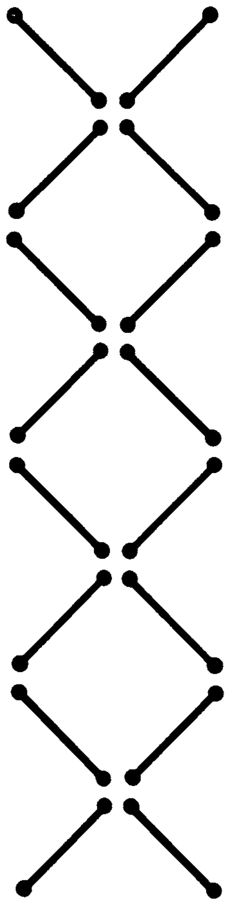


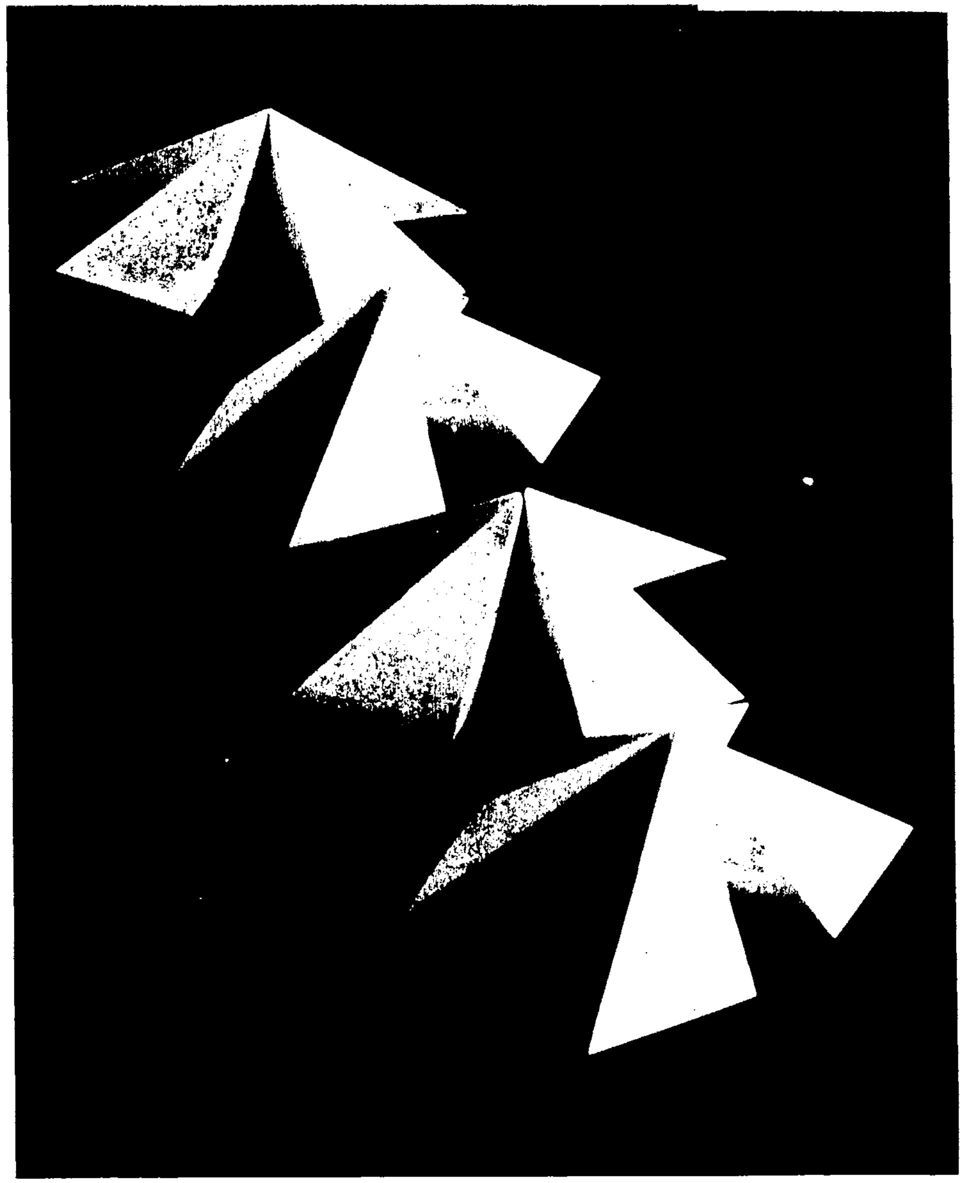




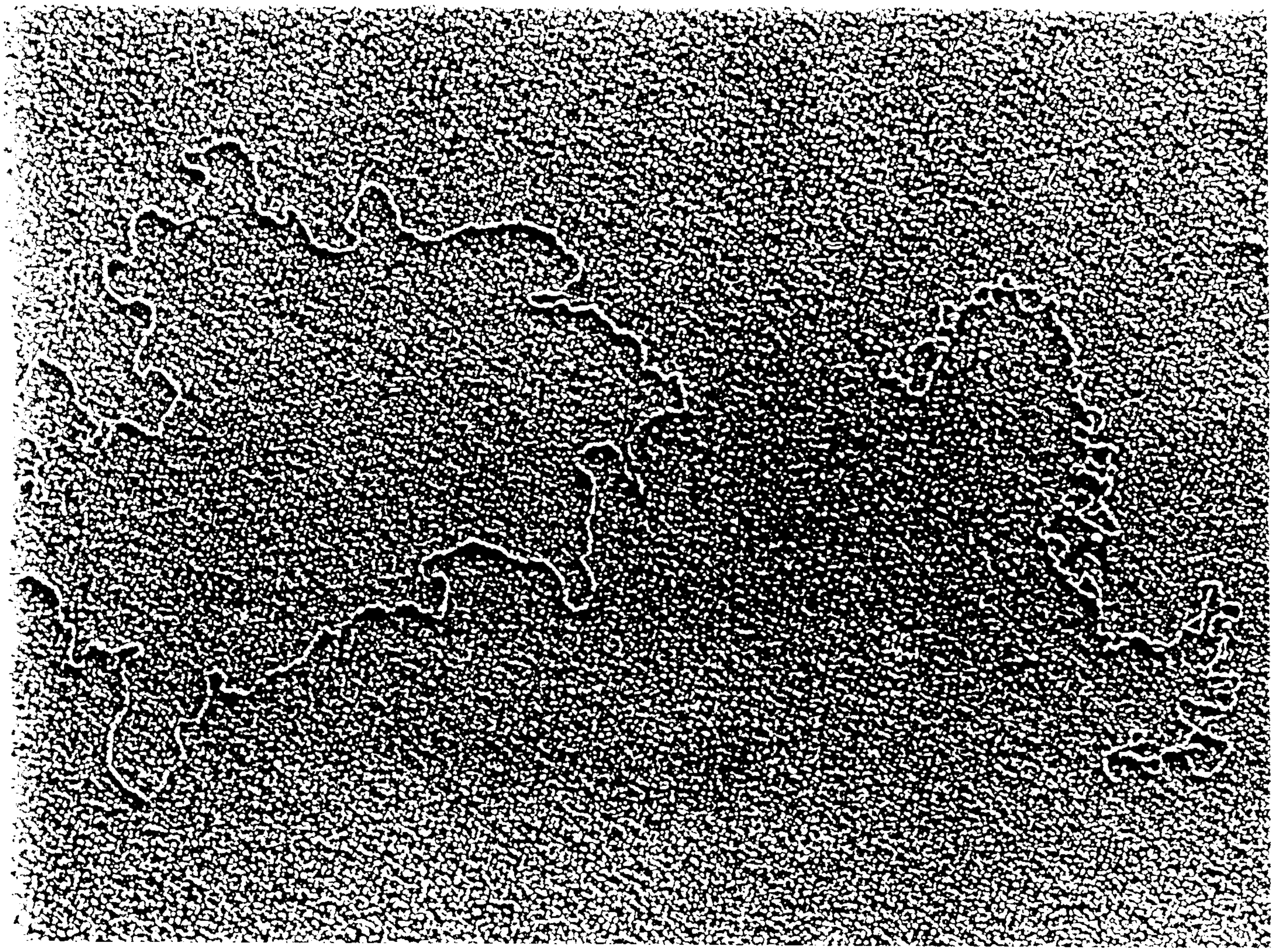


A spontaneous chain disposition.

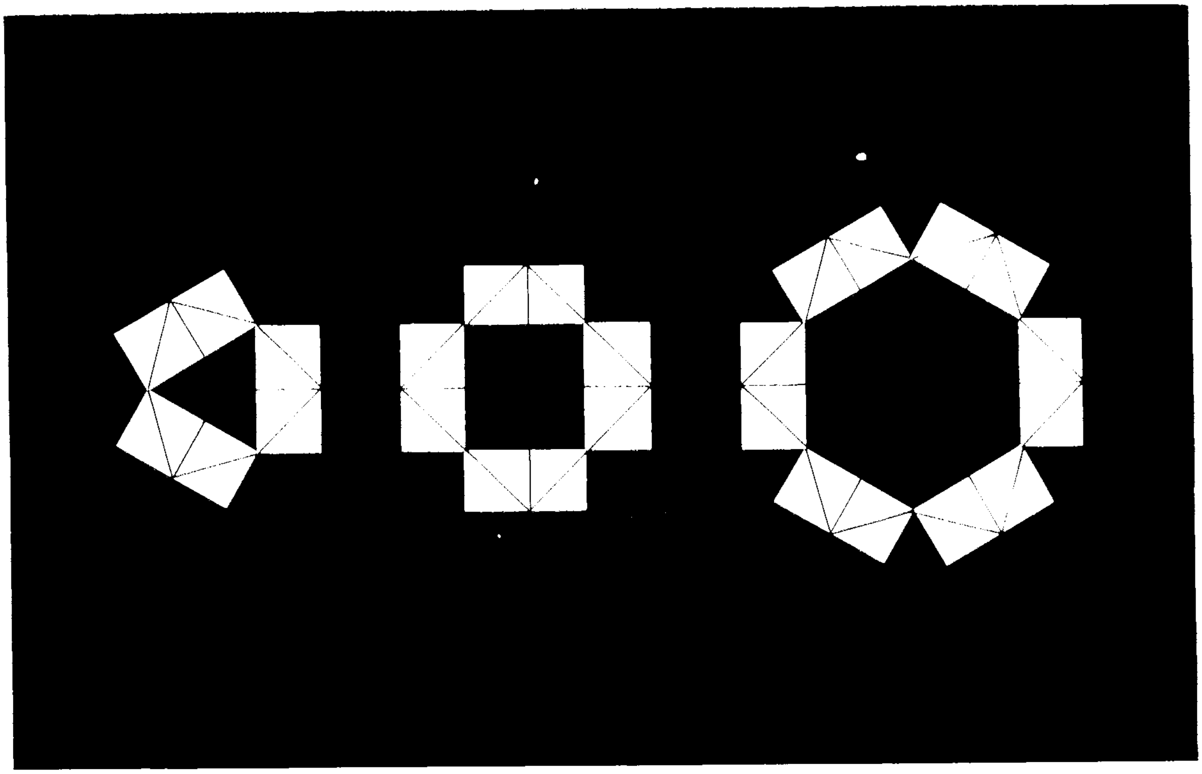




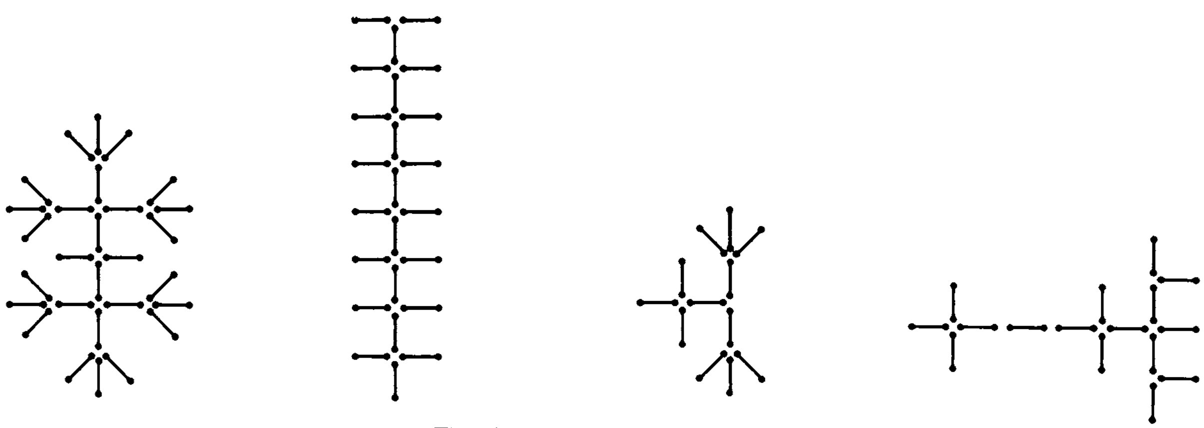
Graphic scheme, seen in top-view, of the joining positions of 16 modules that form, as shown in the photograph of the model, a branched chain.



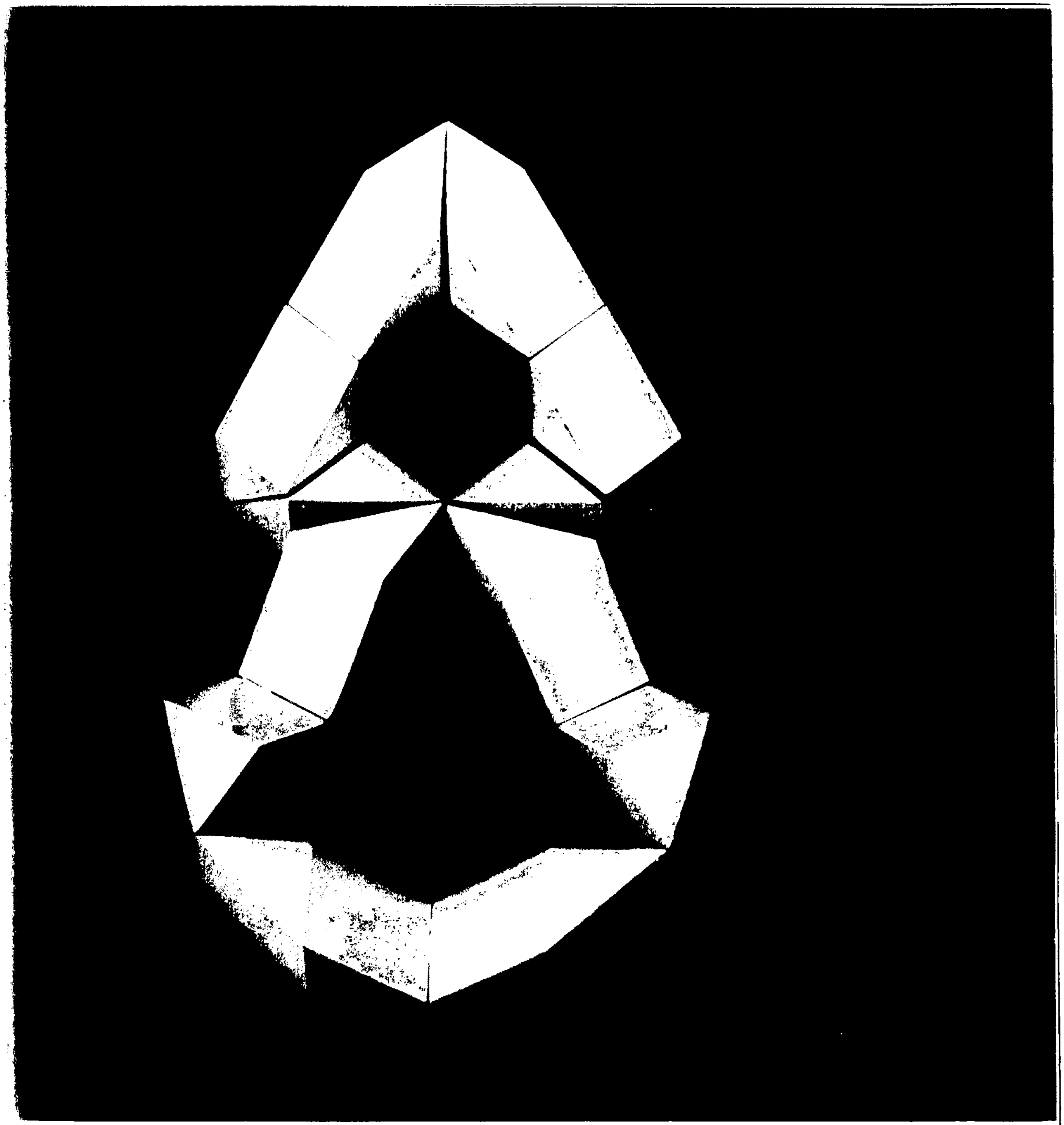
Electronic microscope photograph of a circular DNA molecule.



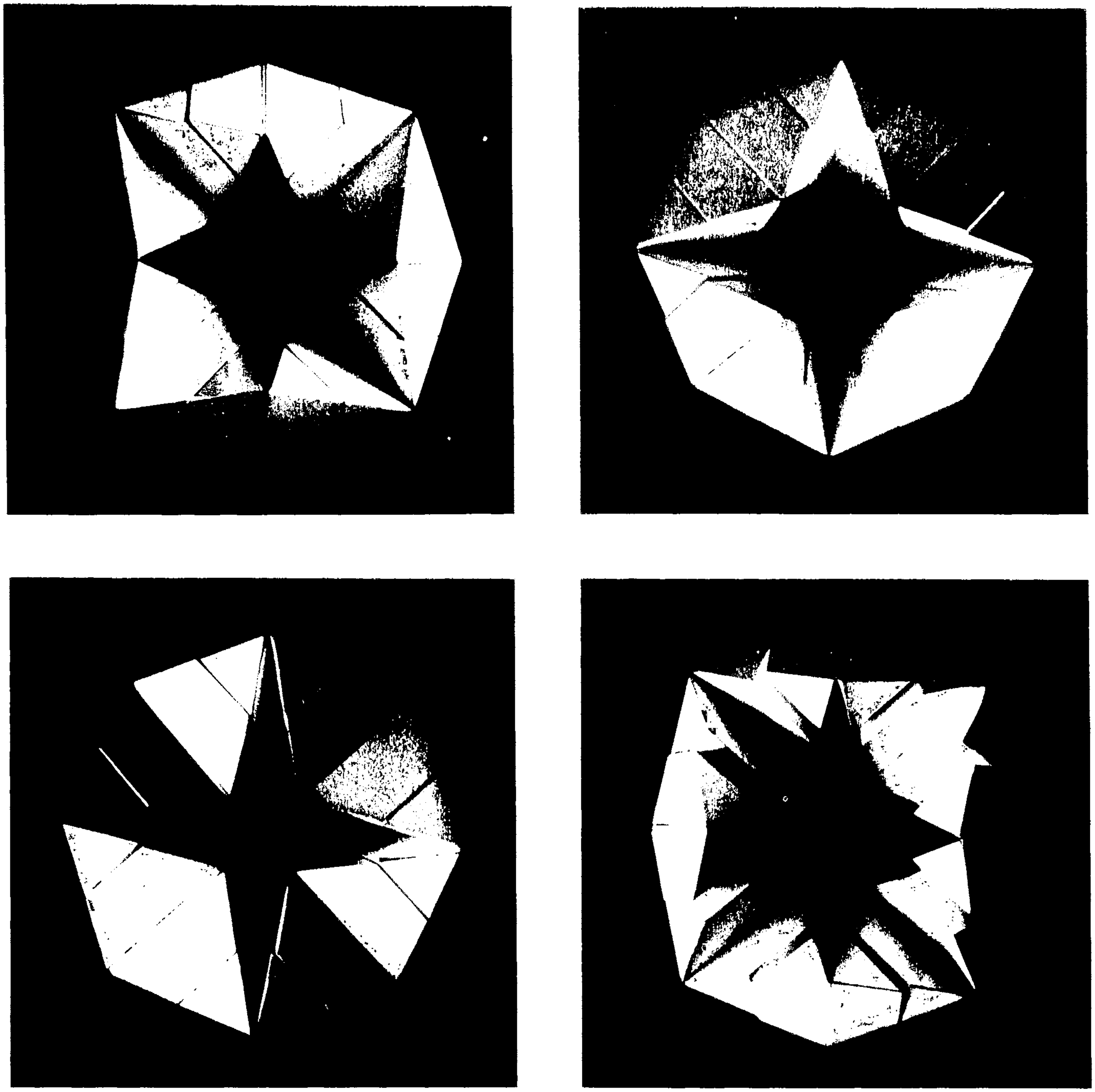
Triangular, square, hexagonal chain, seen in horizontal projection.



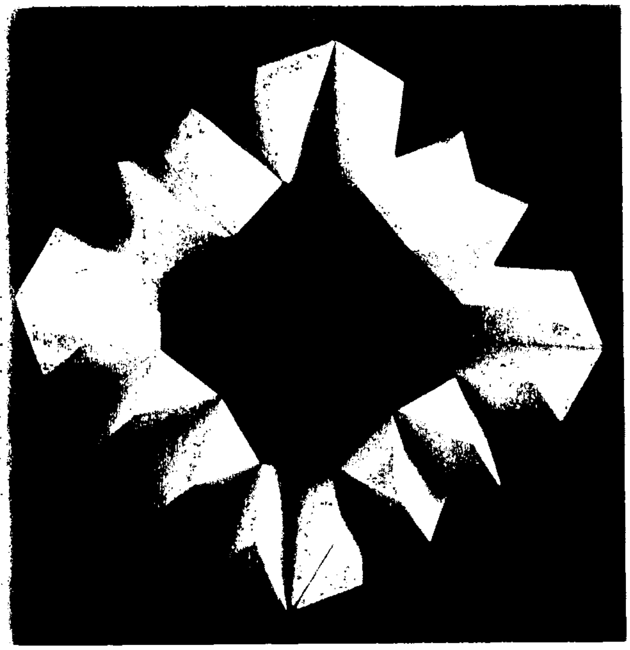
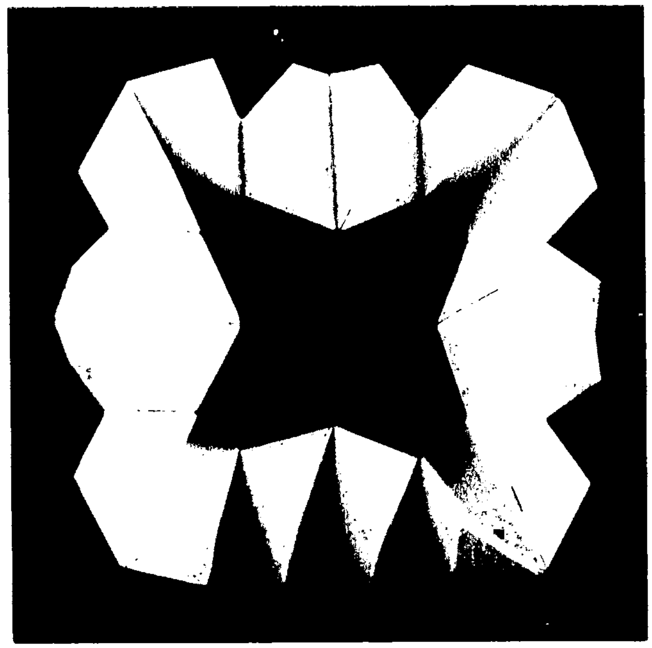
Graphic schema of branched chains. The dots represent the joints.

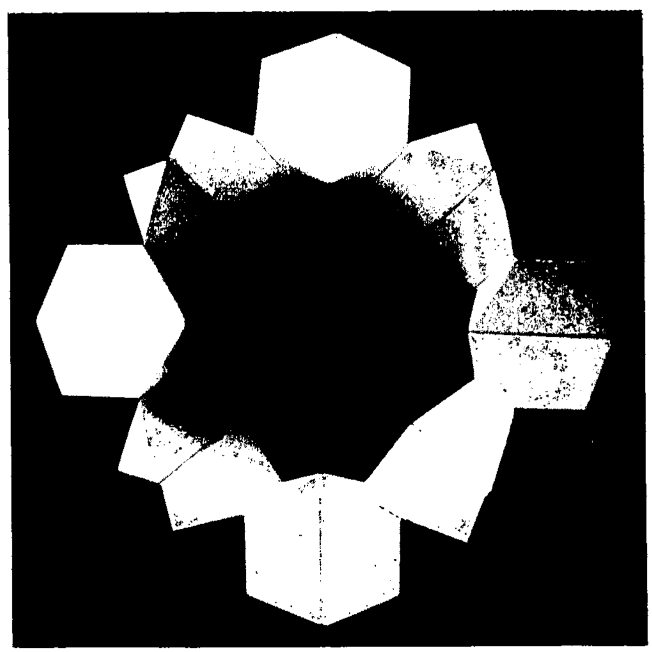


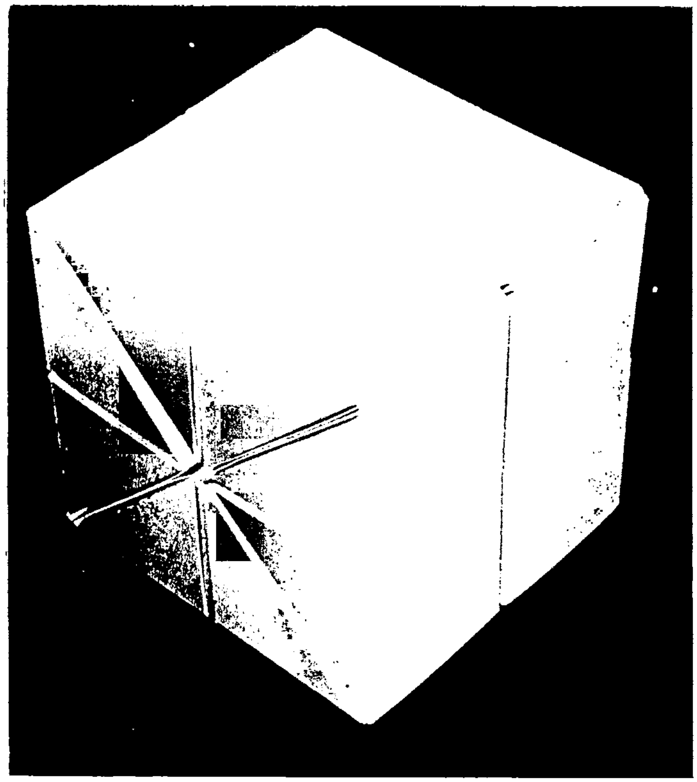
The modules that compose these two triangular configurations, apparently connected together, form in reality a single chain, transformable in many other shapes.



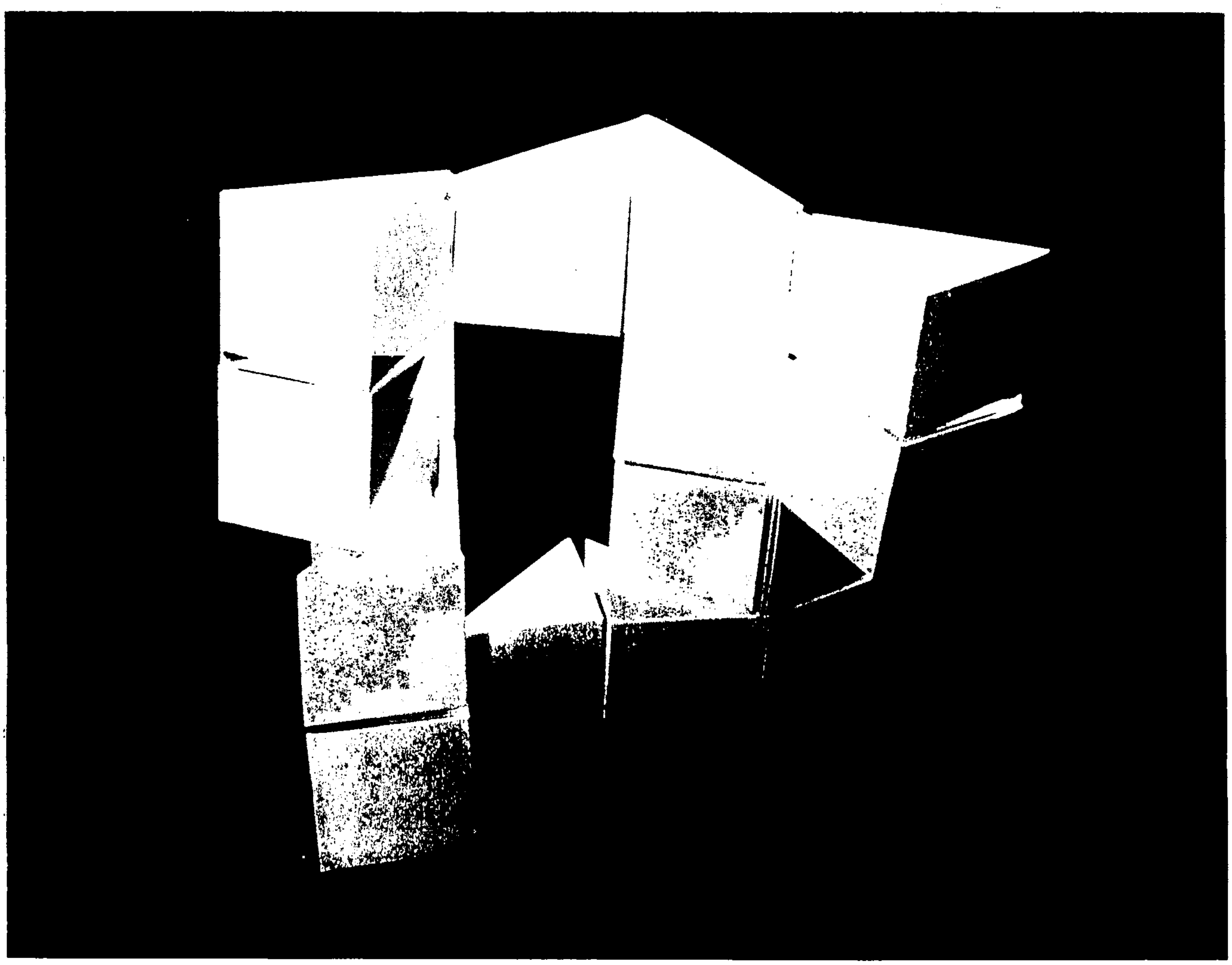
Rotations of 360° of a tetrahedral chain.

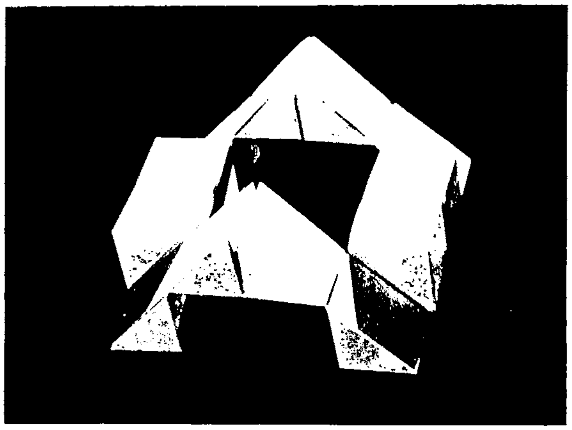
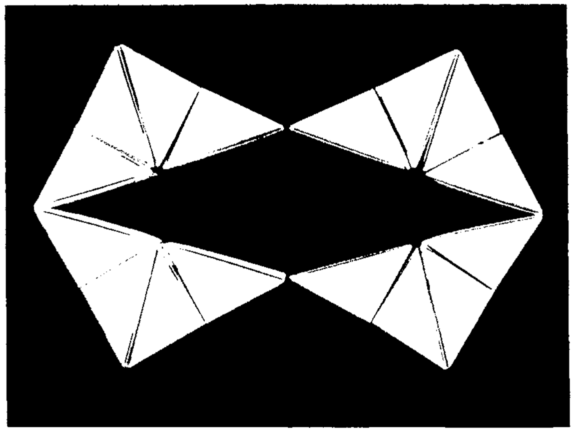
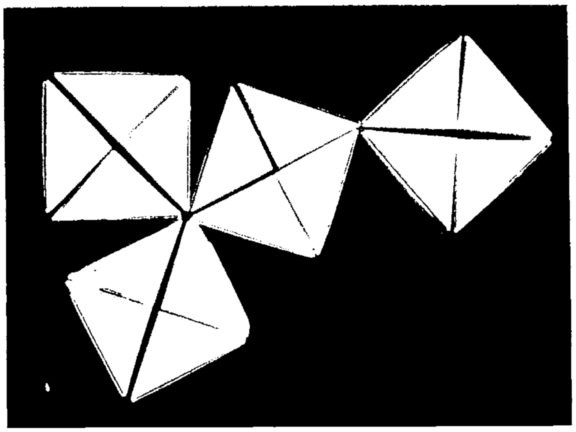
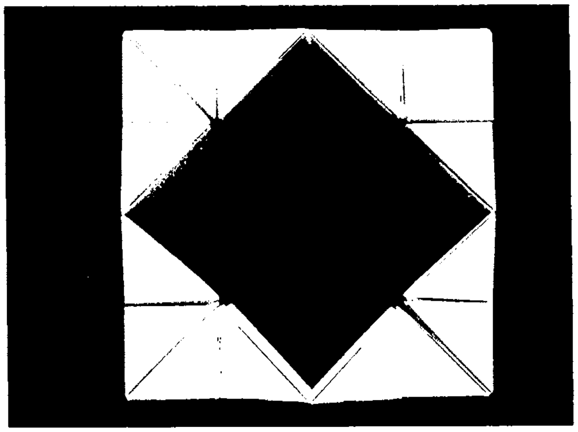
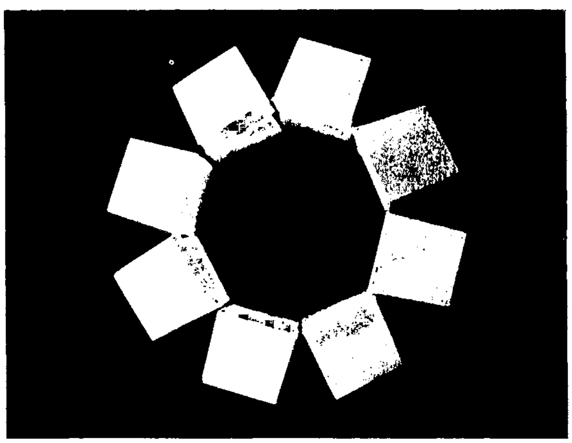
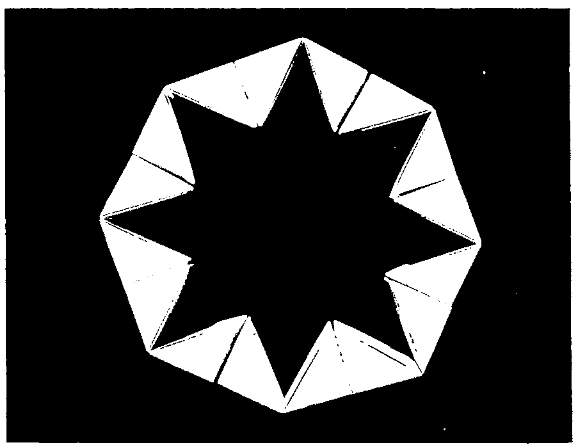
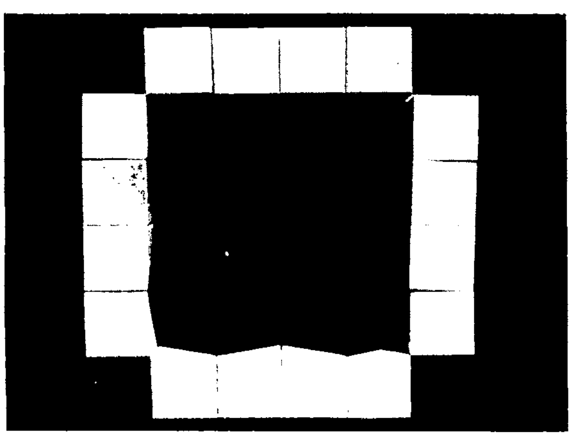
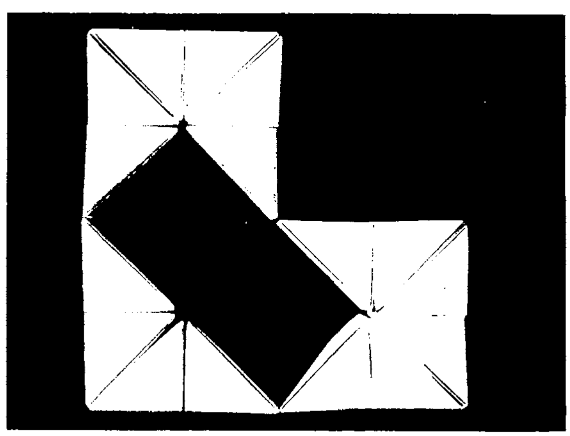



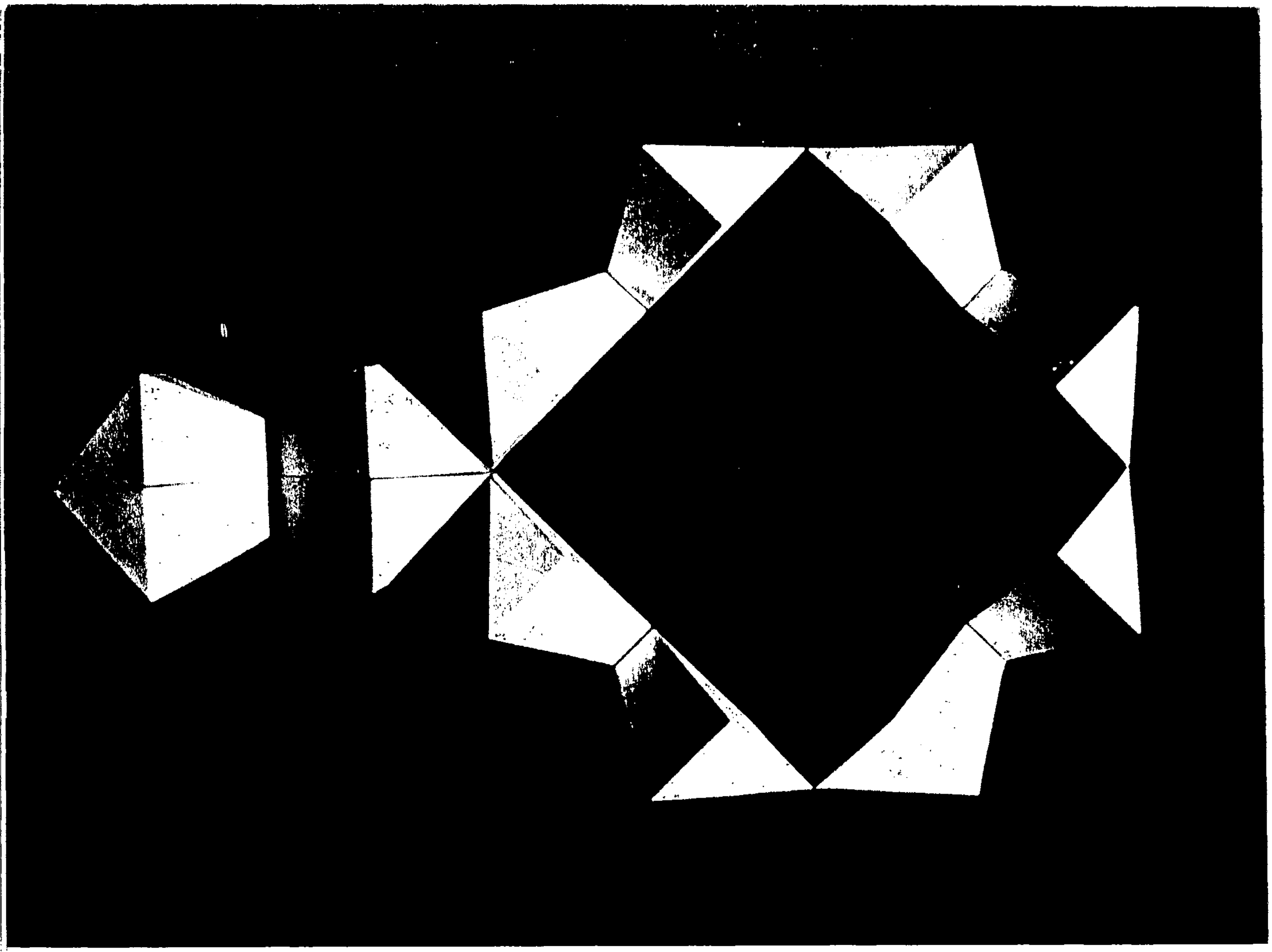


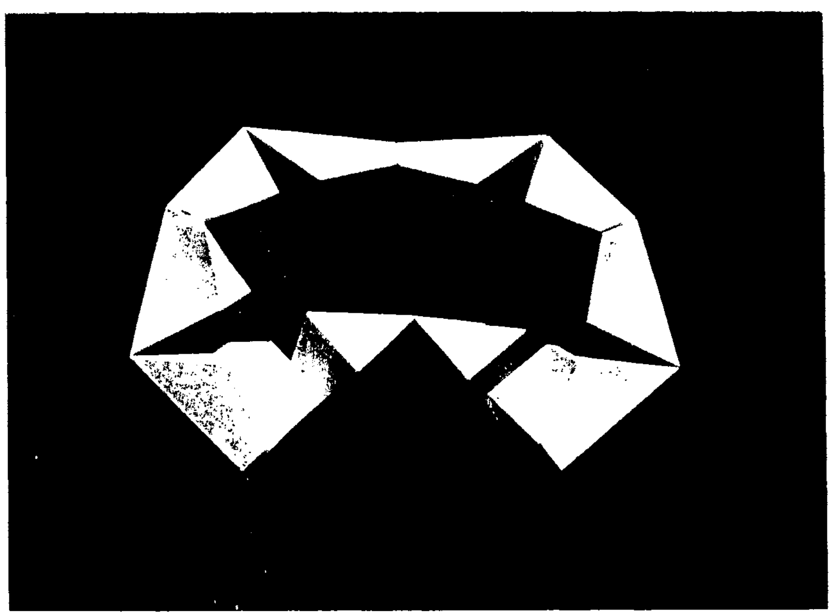


Rotations of hexahedral chains.

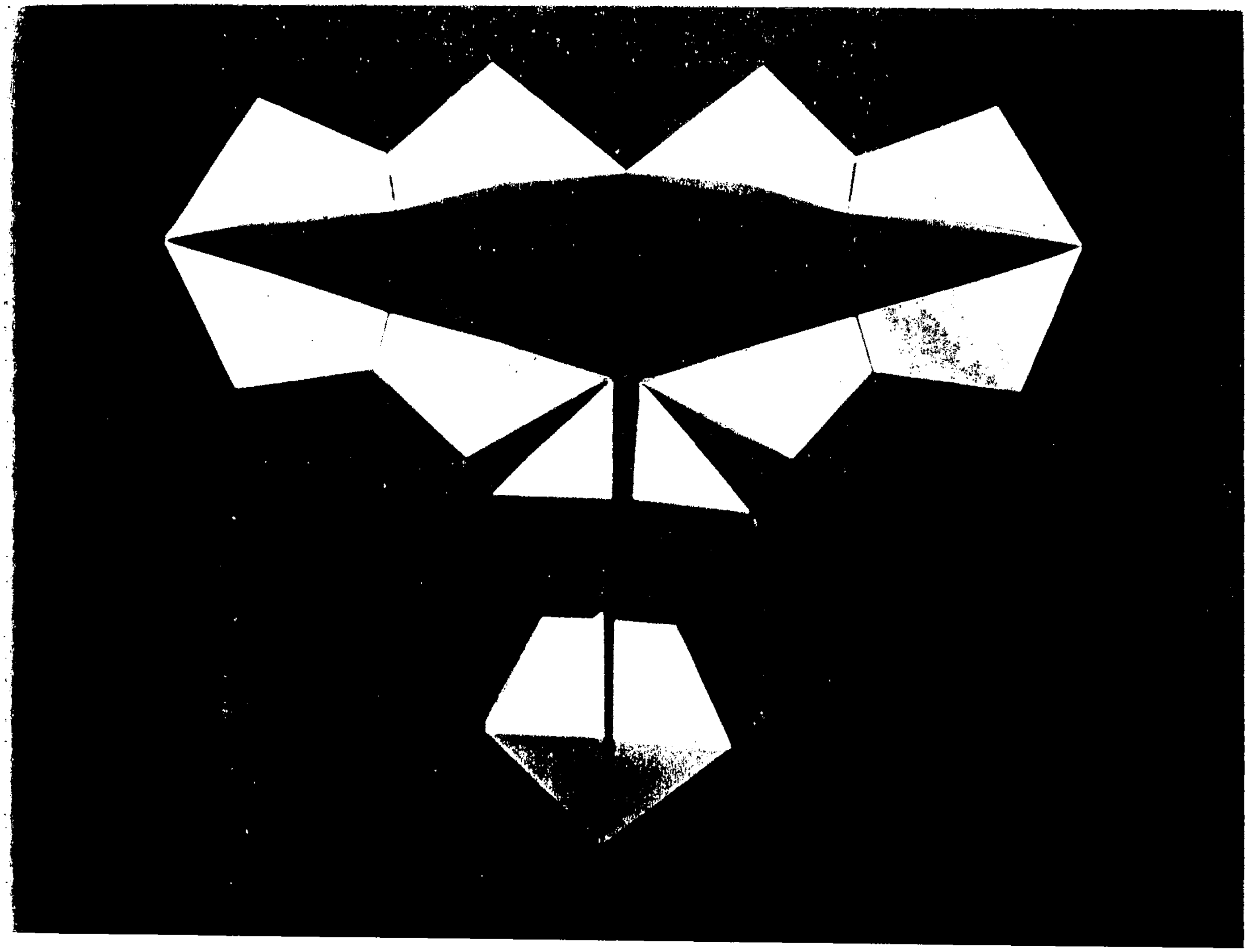


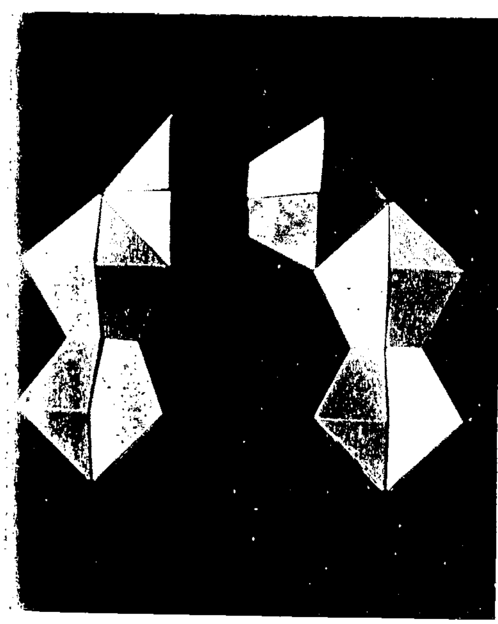









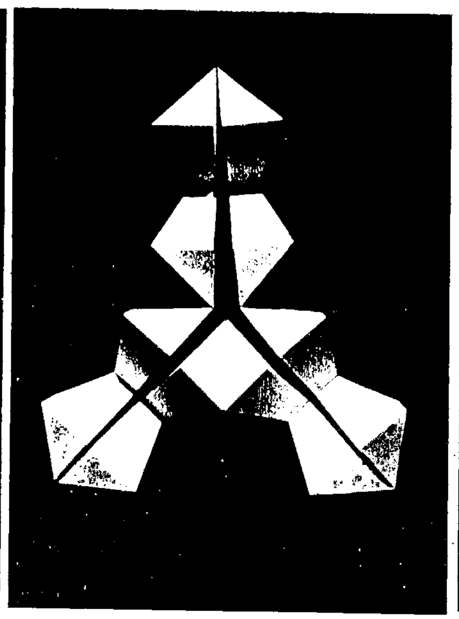


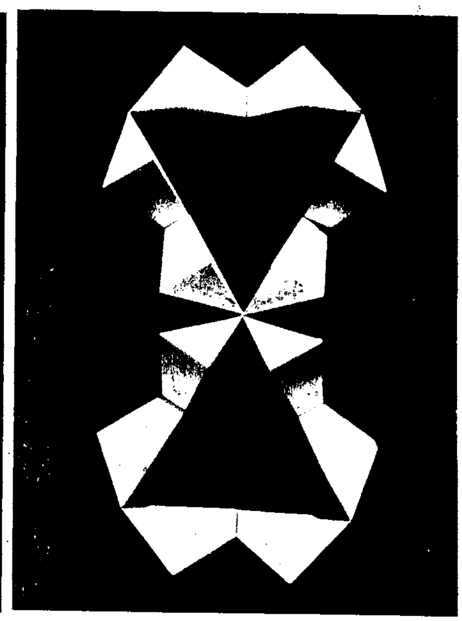


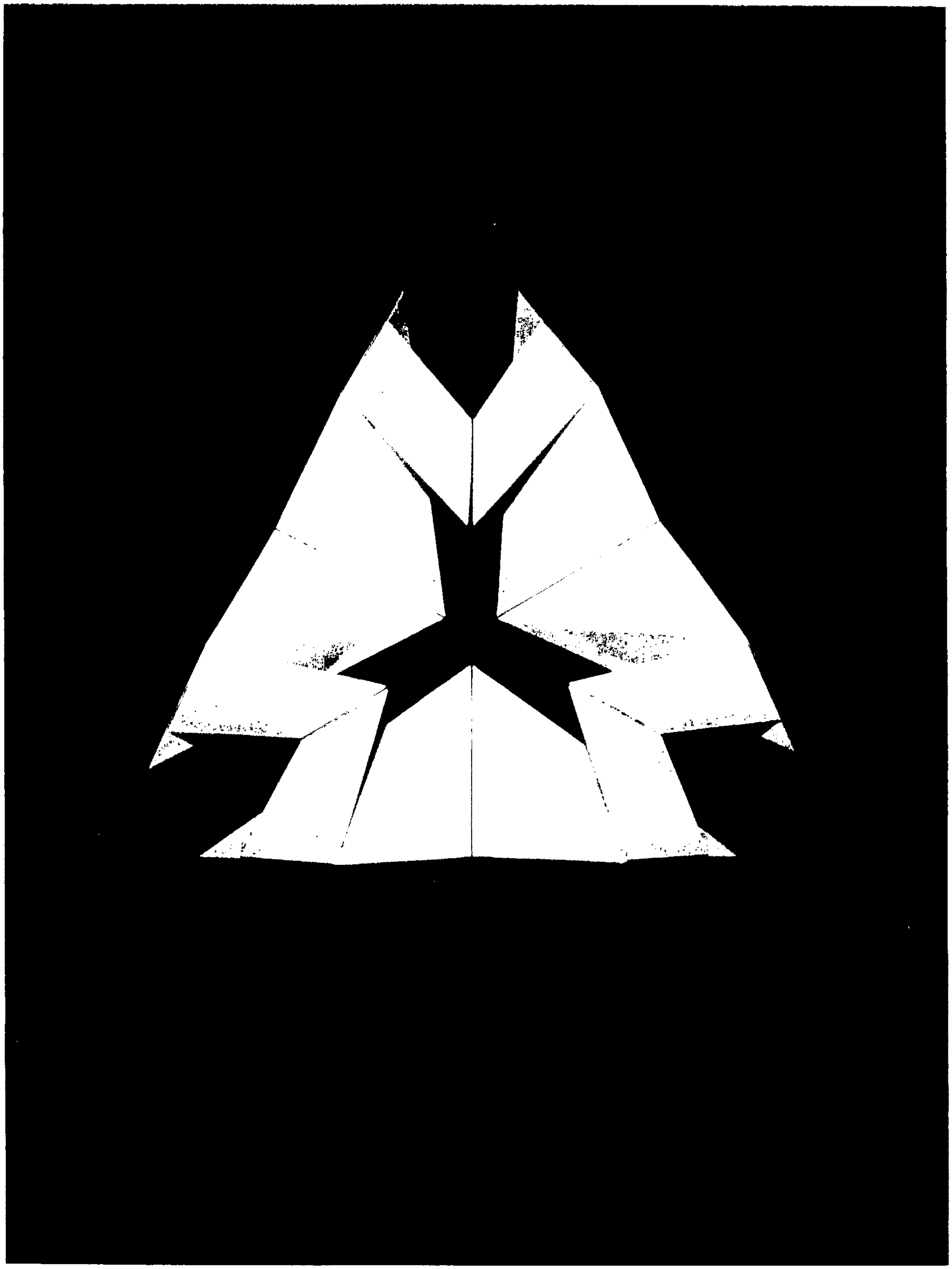
Other rotations of hexahedral chains.

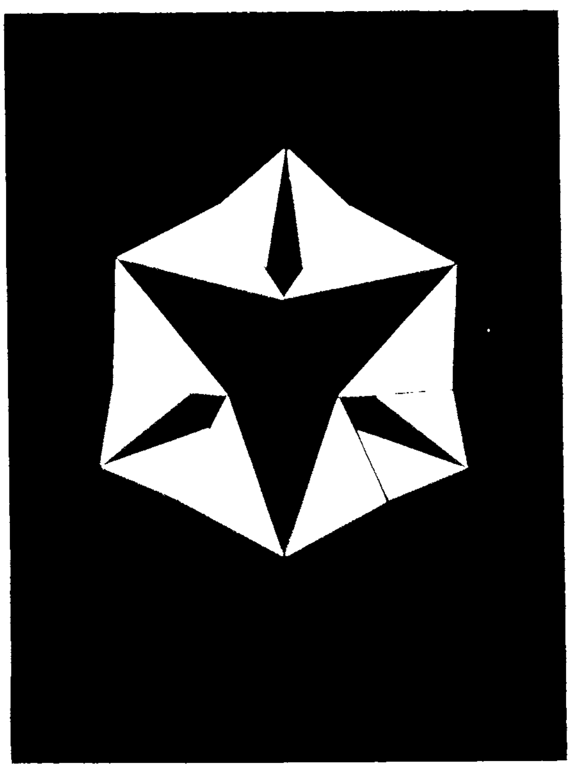








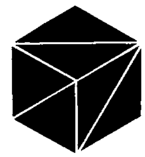




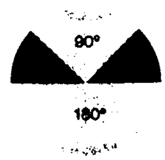
Examples of chains constituted by specular modules and sub-modules.

ANGLES OF ROTATION AROUND THE JOINING AXES

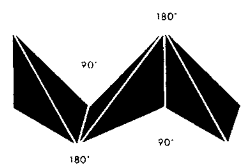
We know that the chains are flexible, and that this flexibility depends primarily on a hinge or bridge that unites two modules into a pair, allowing them to rotate. The repetition of a joint in only one dimension originates an elementary rhythm. Two modules moving one with respect to the other describe a pair of rotations in space. The mono-dimensional repetition of these pairs constitutes an elementary rhythm of rotation. The order, but also a certain disorder, which characterize the transformable chains, is expressed by the combinations of the rhythms of the connections and by the rotation of the modules in different dimensions in space.

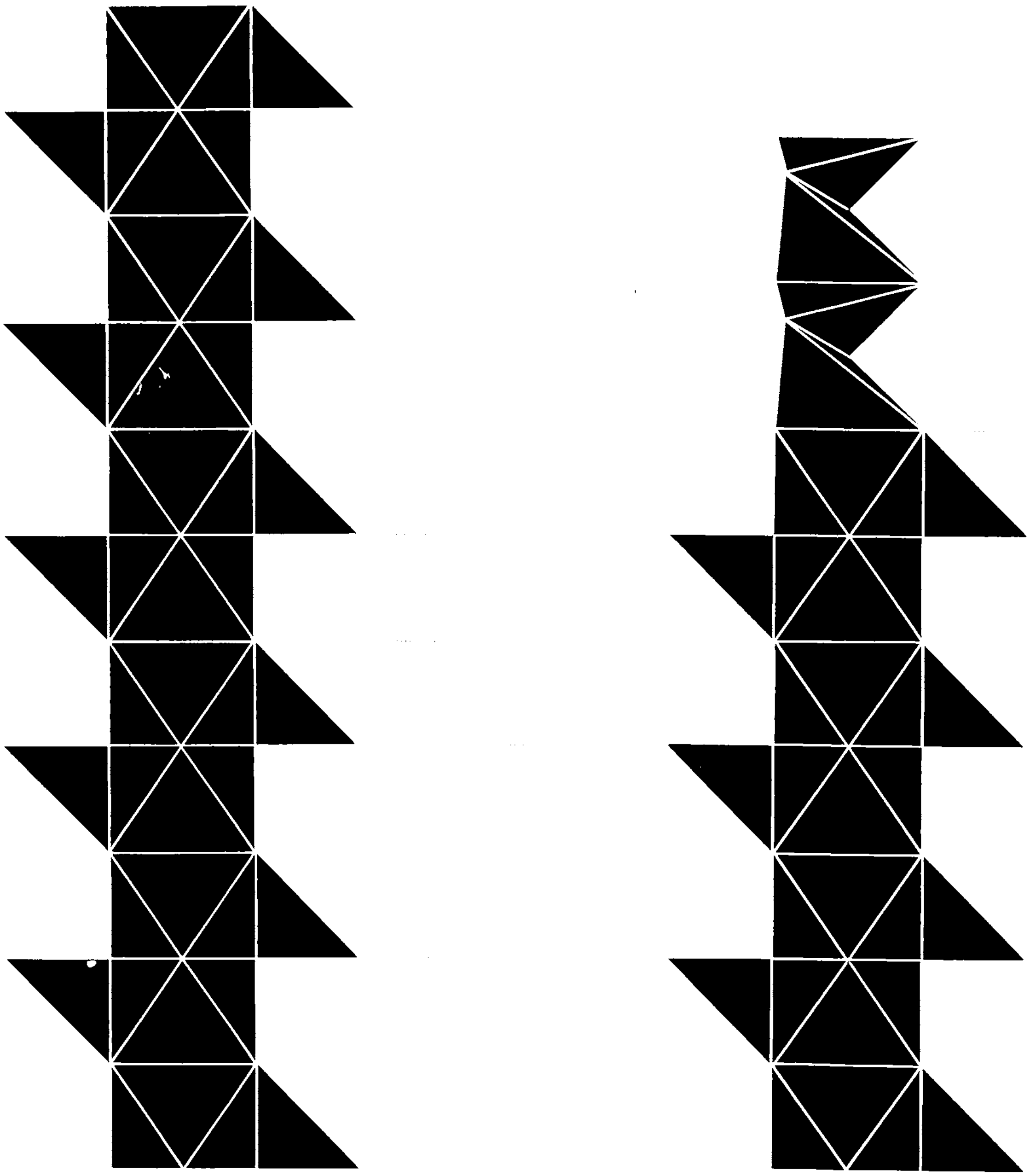


Some examples:
cube constituted by six modules;
three pairs, each constituted by two modular units one specular image of the other, hinged together.

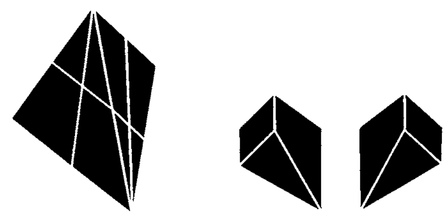


The components of a pair can rotate of 90° and 180° about the flexible connections (these angles refer to the horizontal plane); the rotations are four, two of 90° and two of 180°; the maximum angle of rotation is 270°: a module can rotate about the other clockwise and counter clockwise of 270°. All pairs describe the same angles of rotation; the hinges represent the axes of rotation of the whole chain; the rotations oppose each other rhythmically in two dimensions.

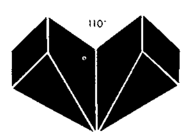




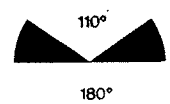
Tetrahedron composed by eight modules;
three pairs, each constituted by two modules, one specular image of the other, joined by a flexible connection;



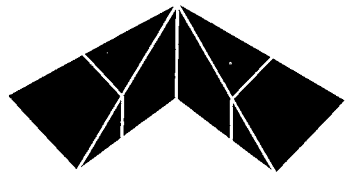
the modular units can rotate about each other with angles of 180° and 110°;
the rotations alternate according to a binary rhythm;



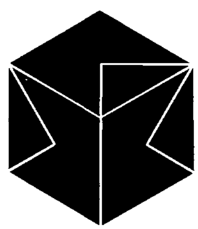
the maximum angle of rotation is 290°.

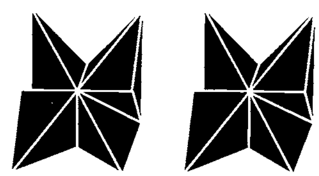


The connections are visualizable in the two dimensions of the dihedal.

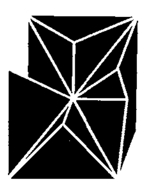


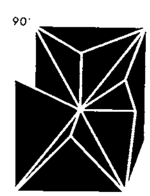
In this case the modules are 12, the pairs are six; the chain is foldable into a form that does not occupy the entire cubic cell;



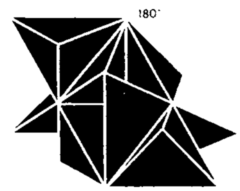


the rotations are of 90°.

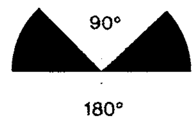




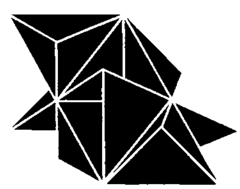
Rotations of 180°.



The maximum angle of rotation is 270°;

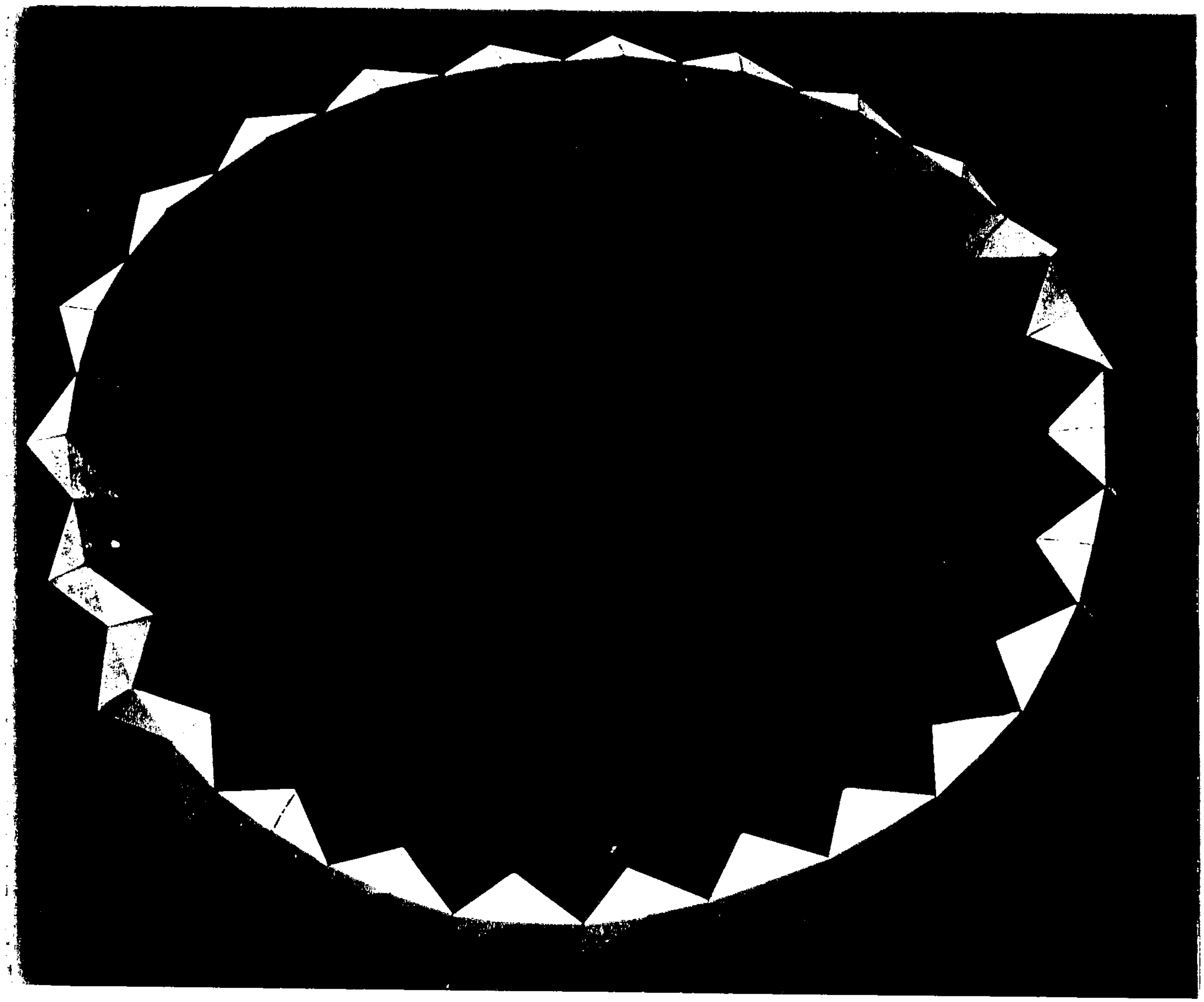


the hinges are in two dimensions.



The rhythm is binary, composed of alternate rotations in two spatial dimensions.

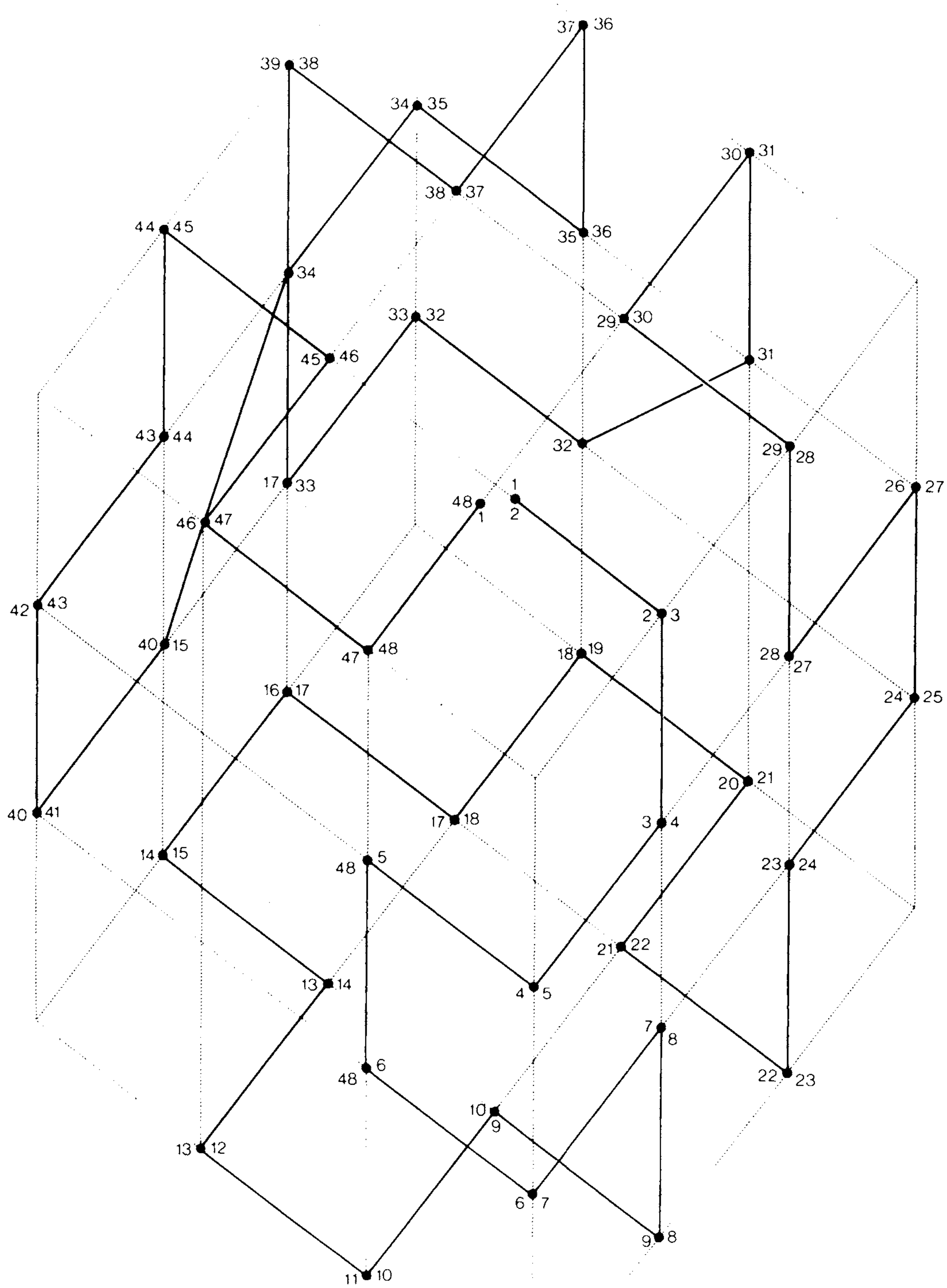
Ring-form chain consisting of 48 specular modules.



STRUCTURING ITINERARY

A module can be constructed in relation to the companion with which it forms the pair. A three-dimensional elementary module consists of two pairs of complementary planes. Two three-dimensional modules form a pair when, being connected together, they adapt to each other; and when the pair of planes which constitute them come into contact following a rotation movement. Spatially, they correspond to each other in a rigorously defined way. We will depart from this elementary condition of correspondence to investigate the relationship of interaction between the modules forming a chain. We will study the interaction mechanisms of the pairs of specular modules as amplification of the mechanism of plane and spatial geometric correspondences of the given initial pair. We will be able, with diagrams and three-dimensional models, to visualize these relationships in the form of structuring itineraries, which are functional to the variation of forms of the chains, and to the comprehension of the variations.

Following this path, it is possible to take 24 modular pairs (48 specular modules) articulated in a ring chain, and have them assume the cubic form. With 48 specular modules, measuring 5 × 5 × 5 centimeters, we will be able to construct, for example, a chain 2.40 meters long, transformable into a cube occupying a space of 10 × 10 × 10 centimeters. The passage from the ring form to the cubic form takes place through rotations which are at times helicoidal progressions, of the modular pairs.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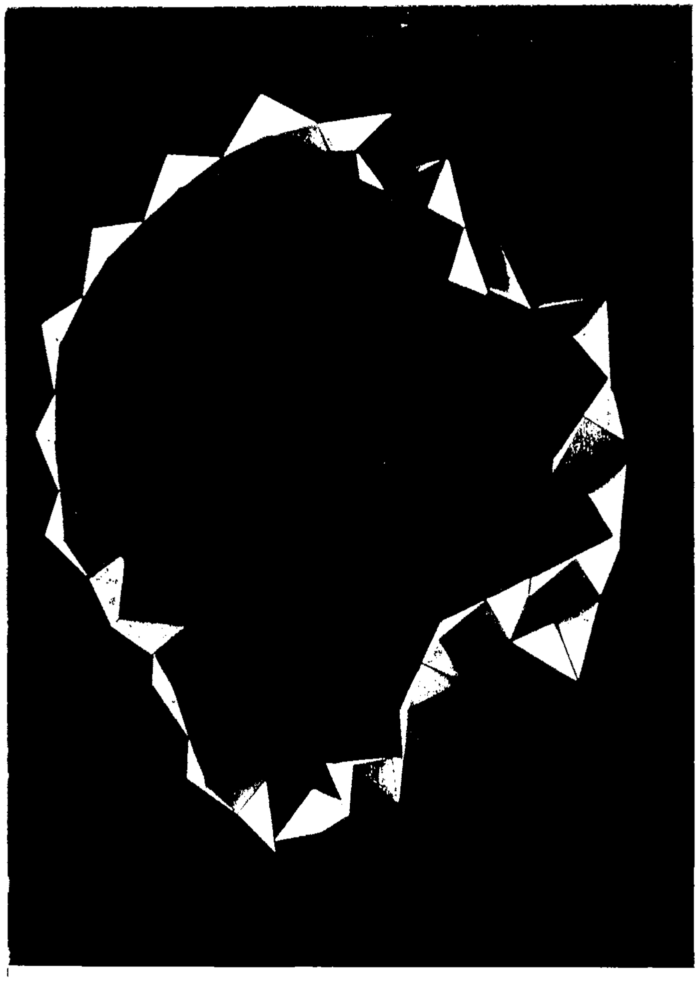
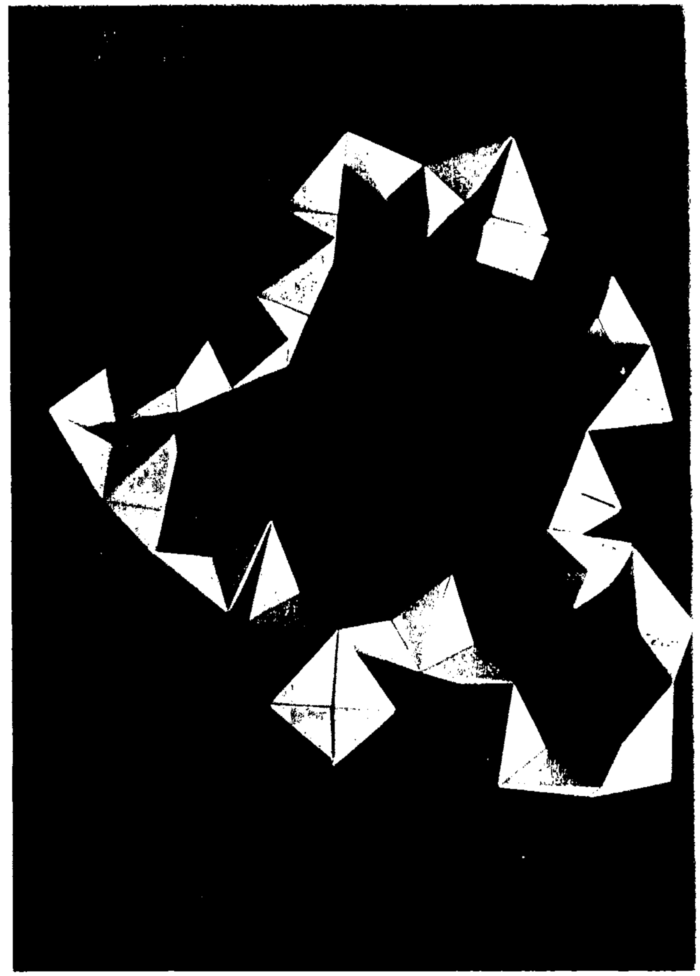
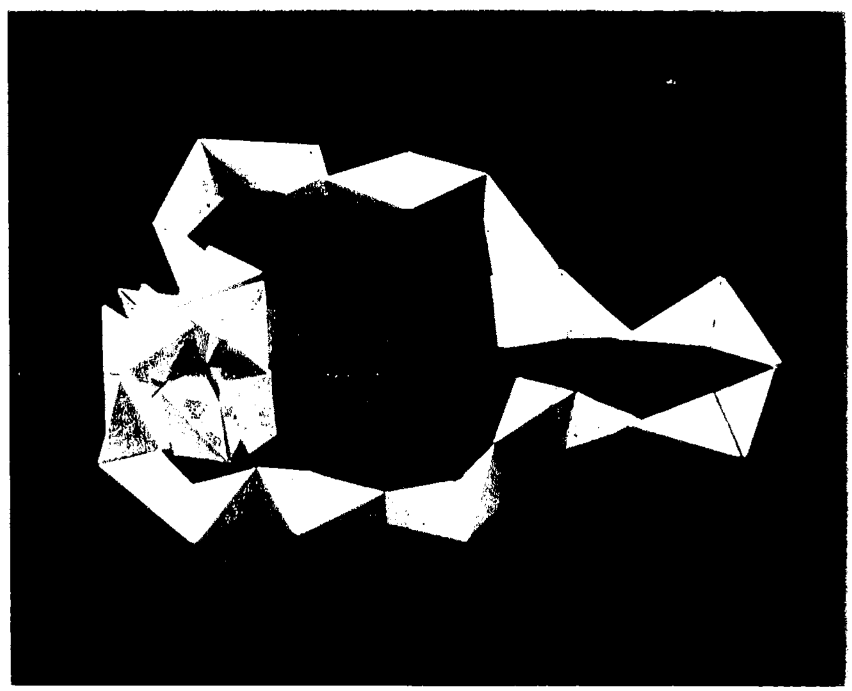
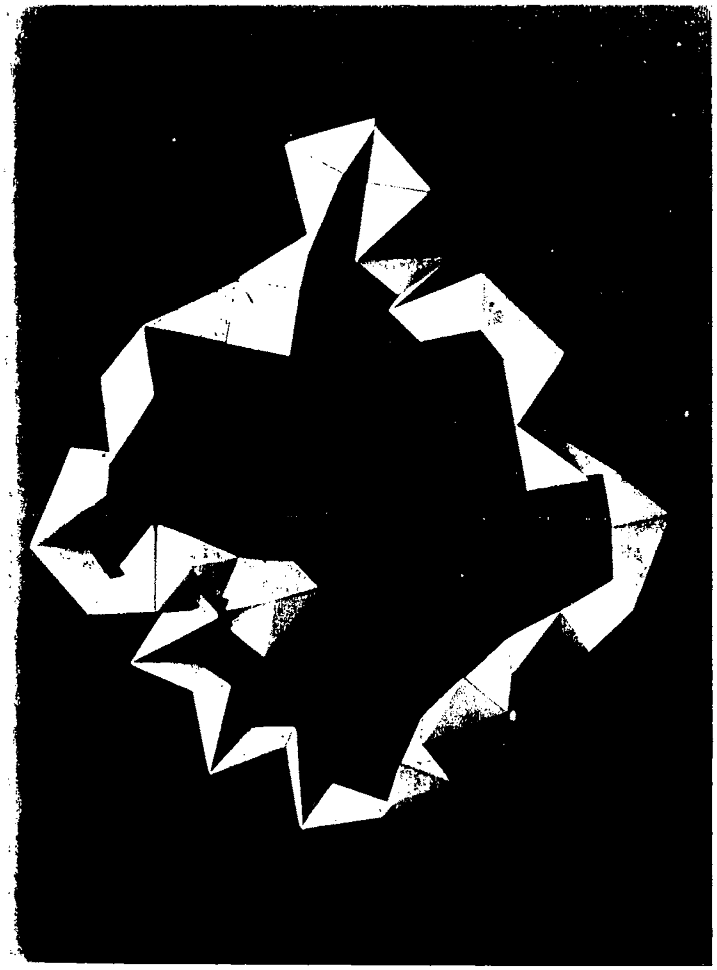
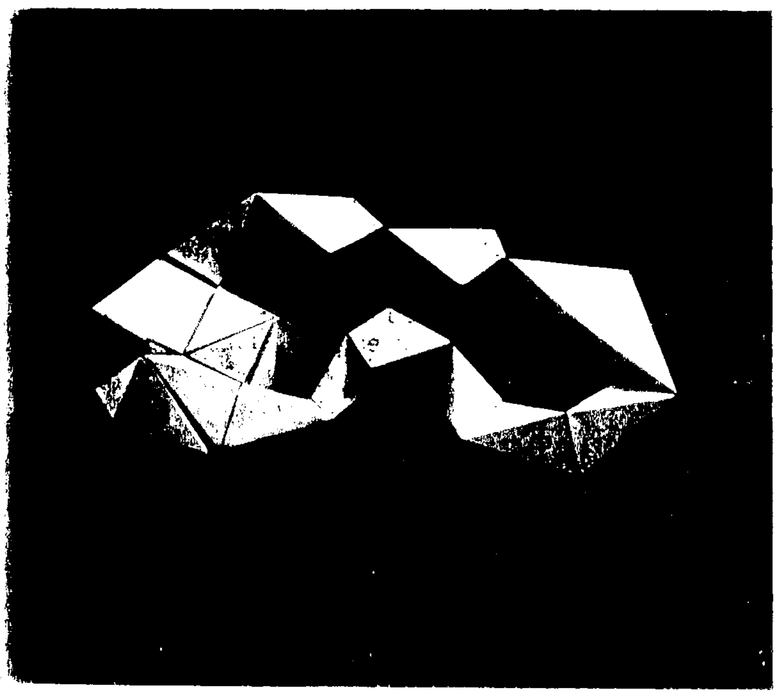
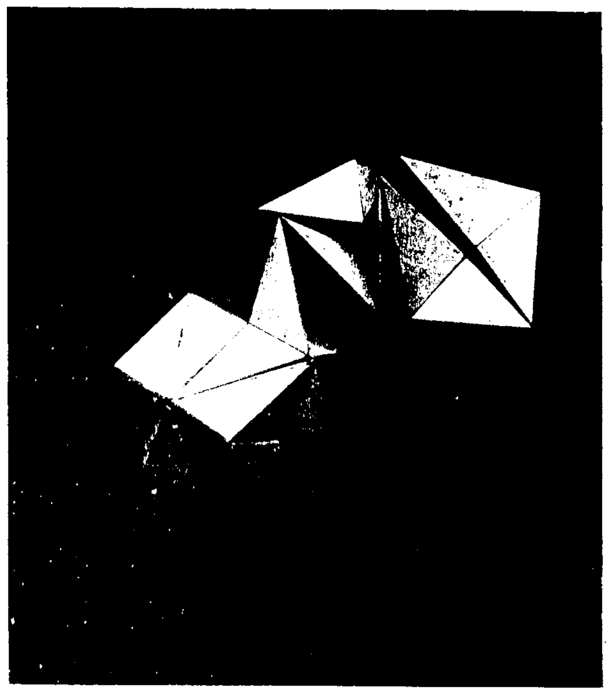


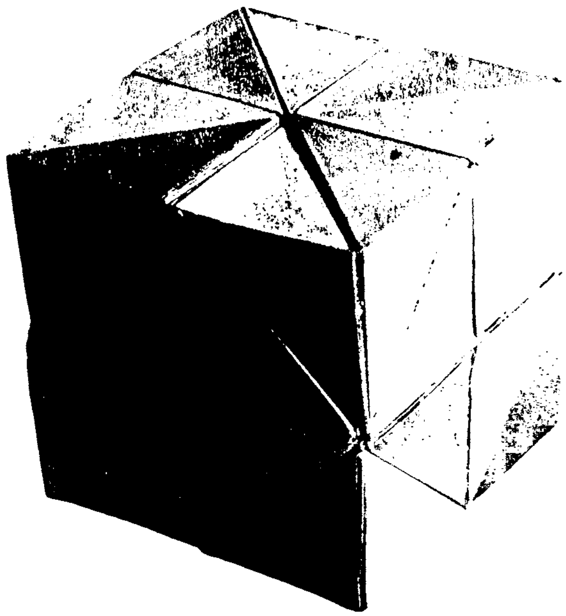












The cubic form is the minumum space occupied by the chain of specular modules seen in the previous pages.

SPATIAL ORDERINGS OF MODULES

A complex structure is the result of combinations of simple structures. We form a pair with two single modules. A chain derives from a combination of a series of pairs of modules. A chain constituted by pairs of modules can originate numerous formal variations, which become more complex if we connect two chains together. With a series of chains connected together in a broader cooperative system we will form spatial orderings of modular chains, etc. This process of complementary connections can continue in an indefinite way. Different levels of modular coordination and articulation are thus formed. The operator must analyze these different variations of structure, and modify, through the selection and the diversification, the components of the combinatory game, in order to make it always more appropriate to the function which it has to serve. From the analysis of a determined series of conbinations of modules, structured in complex configurations, we often gain useful indications about the way to approach further work hypotheses. It is the system of logic correlations through which we learn to mount and dismount each single component of a determined population of modules ordered in a cooperative system, to supply many useful elements of reference to research and experiment new connections. In the chains articulated in spatial cooperative systems, we find that various degrees of "order" and "disorder" are generated by the interaction of the modules following a direct action of some force upon them. To allow the folding of the modules one over the other we will use electromechanical, electronic, electromagnetic, thermal, hydraulic force, etc, or our manual interventions. The variations of form of the chains are determined by the rotation of the modules.

The modules can rotate:

1. following itineraries of which we know the complete map, finalized to the obtainment of a precise formal result;

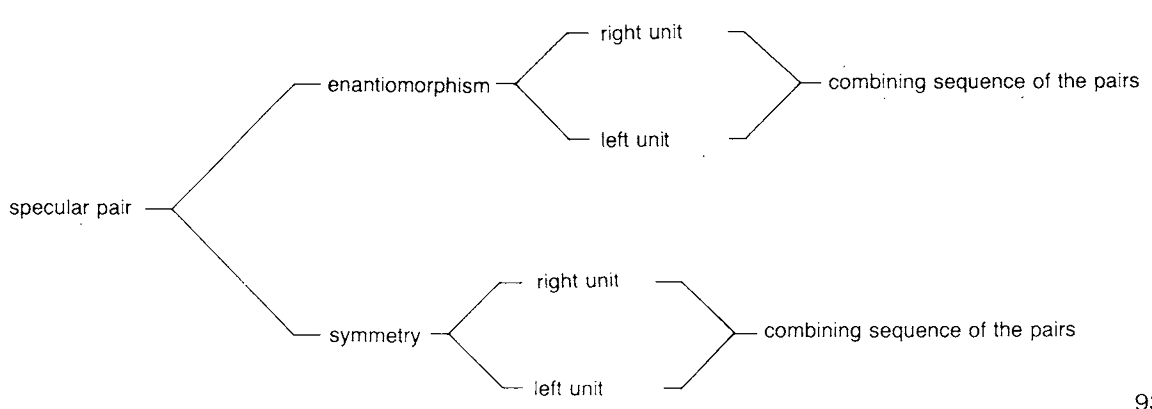
2. along unknown structuring paths.

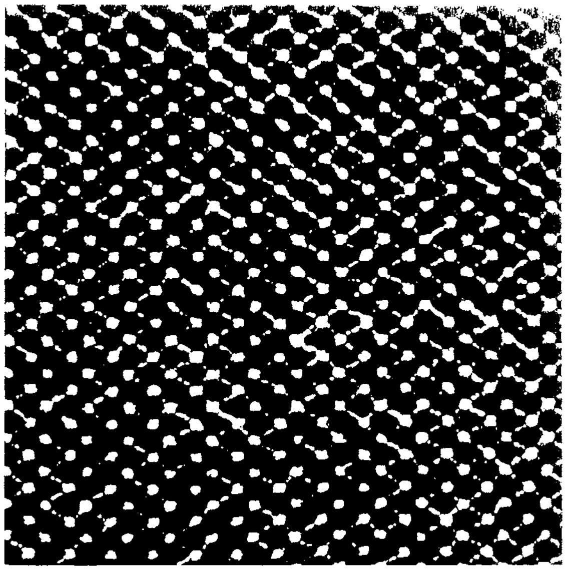
In either case, it will emerge from the modules, in a game of continuous variations, all the forms that the system in which they are coordinated can express.

Manual intervention on an articulated model does not present particular problems when the number of the modules that constitute it is small. The operator can instead have many uncertainties in manipulating a model made of many pieces because of the even radical change that, in comparison to the simpler model, occurs in the system of cooperation of the units that form the articulated structure. The study of the manipulation of the more complex models becomes then necessary, to learn to know the more functional ways of distributing the energy of the manual intervention along the articulated structure. The contructive modalities that allow the realization of an articulated model are based on a concept of feed-back. In that sense, all the elements that constitute it, from the numbers of modules and hinges to their spatial organization, from the

structuring circuits to the angles of rotation, to the modular dynamisms, etc. contribute to characterize it in a way which is partly or fully unpredictable.
With analog procedures to those illustrated so far and others to research and study, with a better knowledge of the pieces that form this game of complementary joints, we can try to articulate and differenciate, always refining it, the formal process that leads to the construction of transformable models.
An experiment which is currently under way, whose goal is the construction of tranformable *fabrics*, regards in the order:
1. The study of the more appropriate materials for the construction of a relevant number of modular units and their relative connections.
2. The study of electronic reticula spatially layed-out in a binary order, with which we can magnetize the modules. All the tranformable models constructed so far present in fact the fundamental characteristic of having the connections (hinges) in two dimensions in space.
3. The study of the interactions of the modules, through mathematical models, to be given to the computer, in such a way that it will be possible to rapidly evaluate various solutions and thus choose those that better adapt to the purpose.
4. The study of diagrams to illustrate the order of each rotation of modular unit so that the whole set of determined series of rotations, thus precise combinations of modules, will result perceptible as *shrinkings, stretchings, wrinklings, vibrations,* etc. In that sense, the electronic paths of the reticula of the module connections, contribute to structure, in a given model, forms designated by chosen linguistic terms.
5. The study of how to intervene, through the use of electronic remote controls, on the articulated models.
To enable the viewer to percept the effects of a modular interaction in a way corresponding to the linguistic terms that designate it, the dimensions of the modules that constitute a tranformable *fabric* will have to be very small. The miniaturization of the modules will have then to be studied as a function of the relationship viewer-object.
In the same way that the construction of a three-dimensional module depends on a precise geometric definition of its fold-out, similarly, also its quantitative organization in space is relative to the ordinatory principles that will give sense to the number of modules. To construct spatial orderings of modules, we will have to know through which passages we will determine their organization.
This sequencial determination must be very precise: we can visualize it through branched diagrams that begin with a type of module and end with the basic type of obtainable combinations (bundles, branchings, rings). The tranformable modular systems themselves, variously combined together according to their organizing schema, will originate more complex spatial orderings. At each stage of each branched scheme, choices must be made regarding the production of modules and their combinations along determined paths. A spatial ordering is to be considered as the combination of a finite number of modules. The type of combination is expressible through an ordered succession of modules. In the same way, branched schema will regard the spatial disposition of the connections (hinges) and their quantitative distribution.

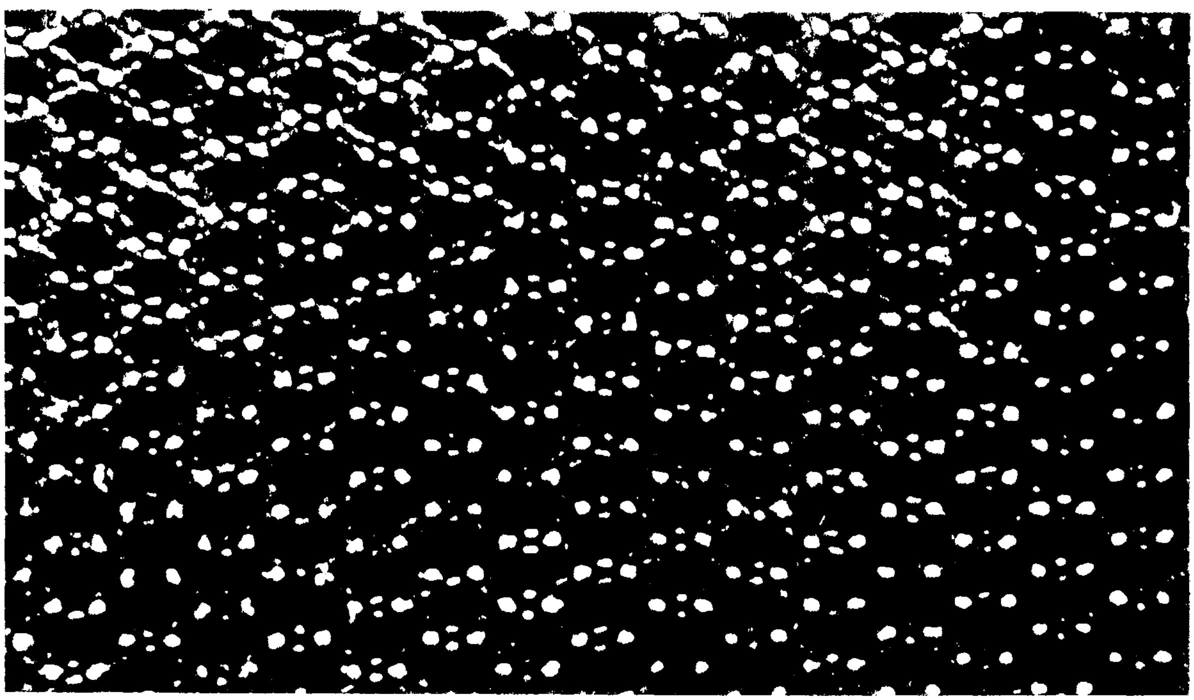
An example follows.



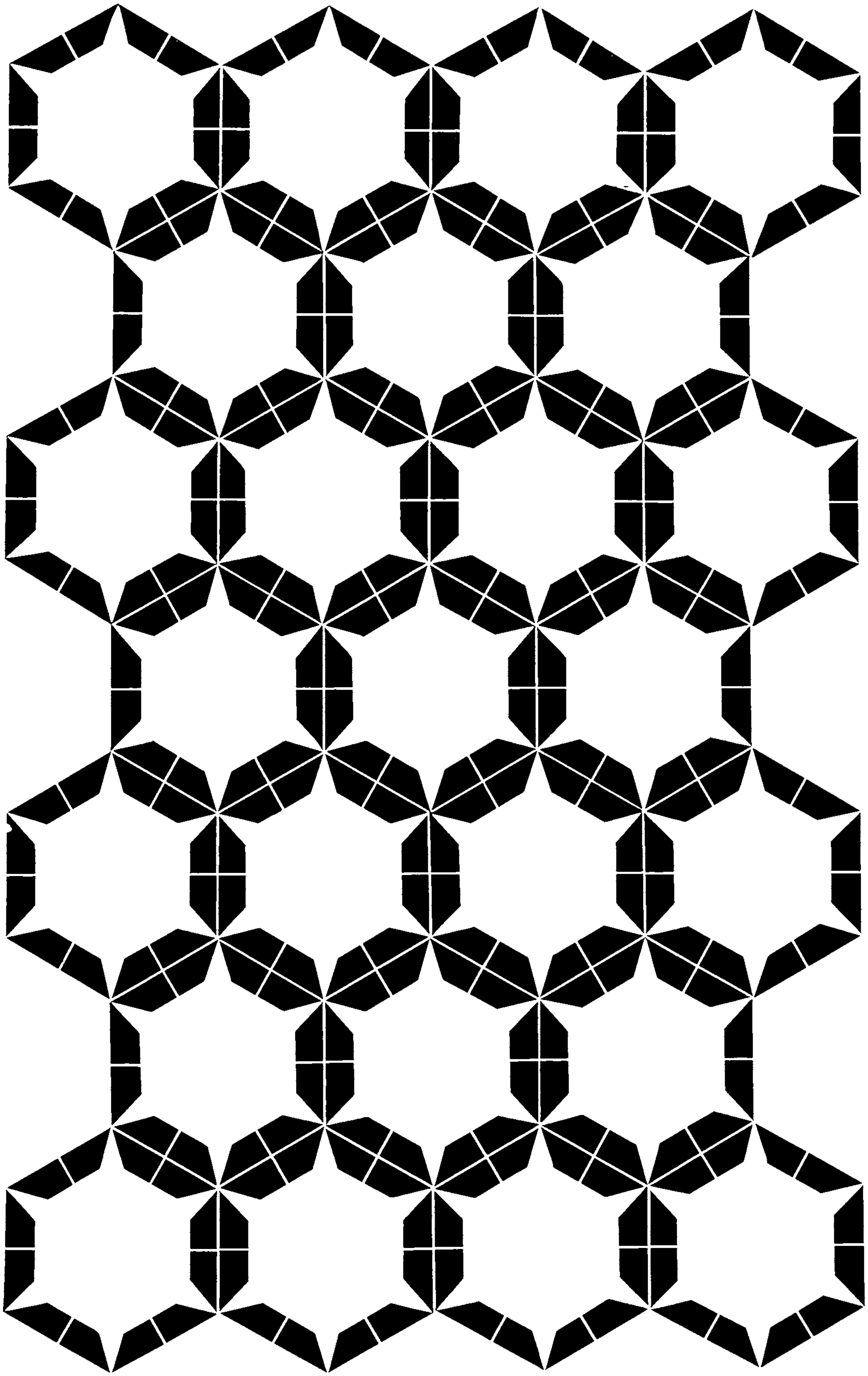


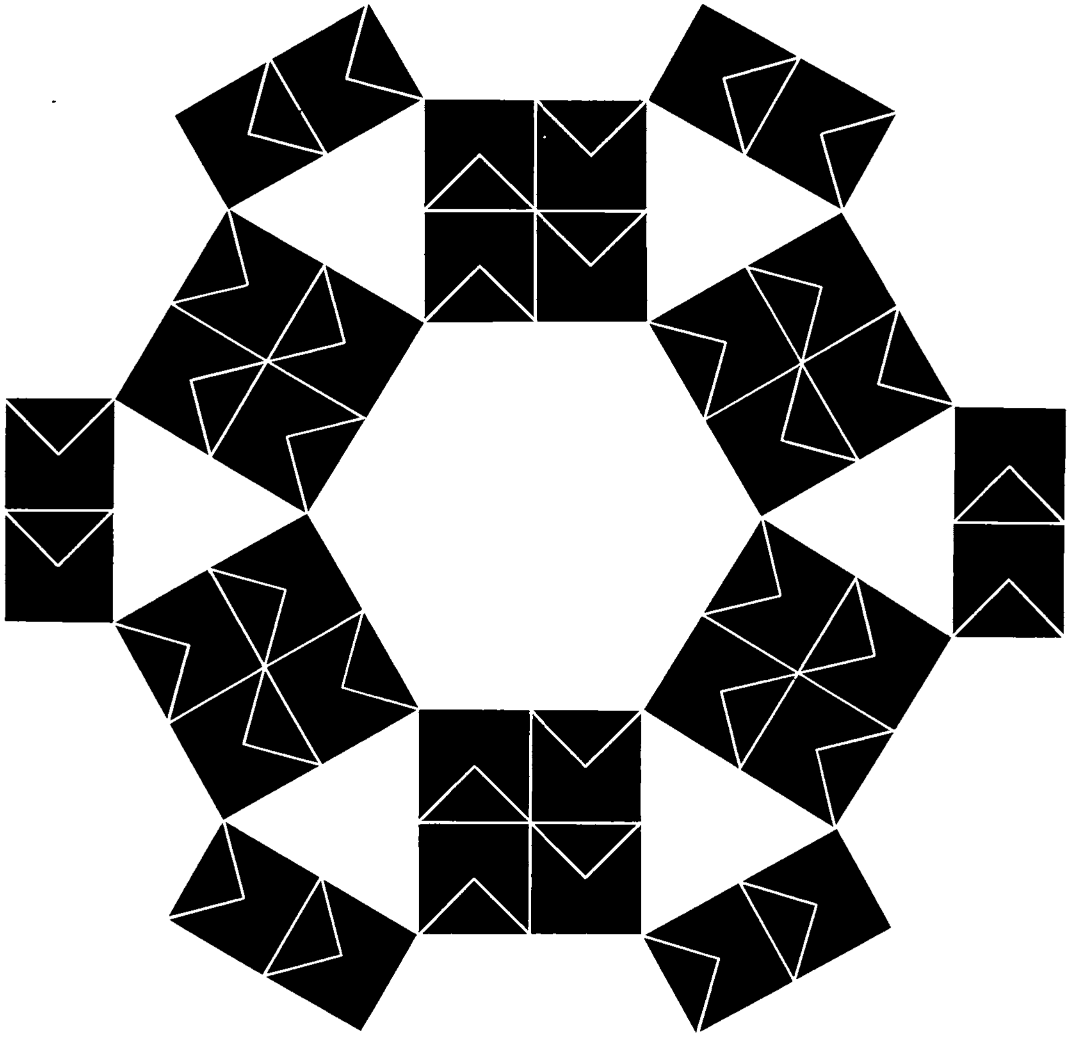
In the front page, 25 ring chains, 150 pairs, 300 modules, form this spatial ordering seen in horizontal projection.

Crystal of thropomyosin. The microphotograph is a projection on a plane of a three-dimensional reticulum constituted by molecular filaments connected across and enlarged 200,000 times.

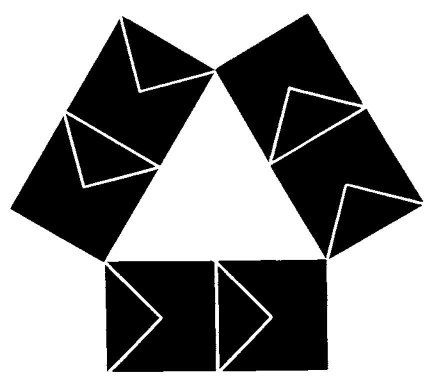


Microphotograph of a double-diàmond shaped net of a muscular fiber.





Top view of a group of chains with a triangular closed boundary, connected together.



Six chains equal to this one form the whole.

The purpose is:

1. to render geometrically determinable the relations that intercede between the parts that constitute the module;

2. to consider the aspects of cyclicity, harmonic growth, to be found in the operations of rotatory symmetry, executable on the plane, as a function of the three-dimensional configuration of the module and its repeatability;

3. to construct chains which are constituted by pairs of modules, in which one of the components is the specular image of the other (modular pair: symmetric and enantiomorphic);

4. to consider the tetrahedron, the cube, the octahedron, the dodecahedron, the icosahedron and all the various families of polihedrons as form-cells determined by foldings, along precise structuring itineraries of chains of modules, ordered in linear sequences;

5. to make visible the form of the itineraries that the modules determine while changing position in relation to the transformability of the system to which they belong;

6. to research, among the structuring itineraries of the various transformable modular systems, those which are most economical, in the advent of the possibility of using, through these, electric, magnetic and mechanical forces;

7. to investigate the aspects that differenciate a tranformable system from the other, in terms of plane and spatial geometry;

8. to point out that the number, the type of modules and the order in which they are laid out, represent that which singularly characterizes the various modular systems;

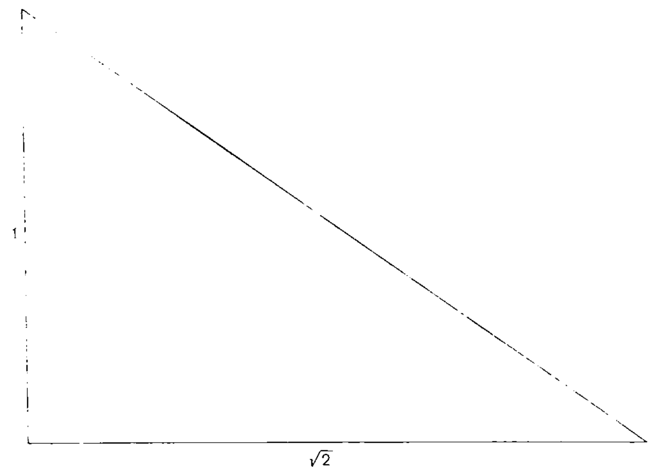
9. to research new combinatory and structural components, organizing data regarding spatial orderings of modular systems, which can be transformed into mathematical models for the programming of the computer;

10. moreover, we believe that the manual intervention (to which the time component is related) is the fundamental and simplest way to act in a tranformable sense on every system of modules articulated in a chain.

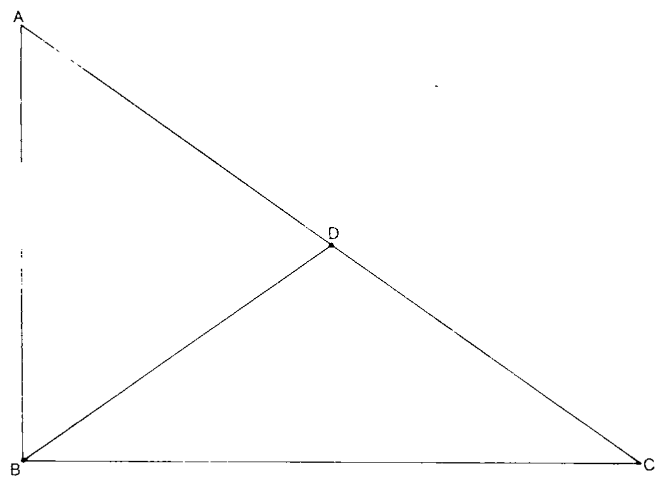
In every chain exists a constant relationship between the number of modules and the number of connections. If we raise the first, we must raise the second, the degree of flexibility within the modular system grows, while in the case of models of larger dimensions, the coordination of manual interventions becomes more complex and the "disorder" related to the combinatory game of the modules is also larger. An example could be a series of chains of various form constituted by modules in periodic progression (96, 768, 6144... 48, 384, 3072... 192, 1536, 12288... etc.). In any case, no matter what the dimensions are, we can say that in a transformable system, the manual or otherwise intervention determines various and different adjustments of the modules in many spatial directions: with the term *stereomodular* we can define the organization of that model in which its modular units synchronously cooperate to the pluri-dimensional transformability of the system which they constitute.

THE DIMENSIONS OF THE FIVE REGULAR POLYHEDRA

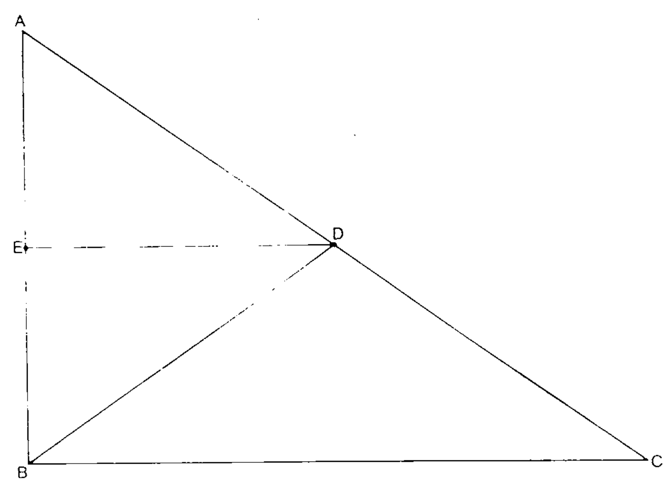
1. Construct the rectangular triangle $1 \cdot \sqrt{2}$ on which the fundamental dimensions of the five Plato polyhedra will be defined,



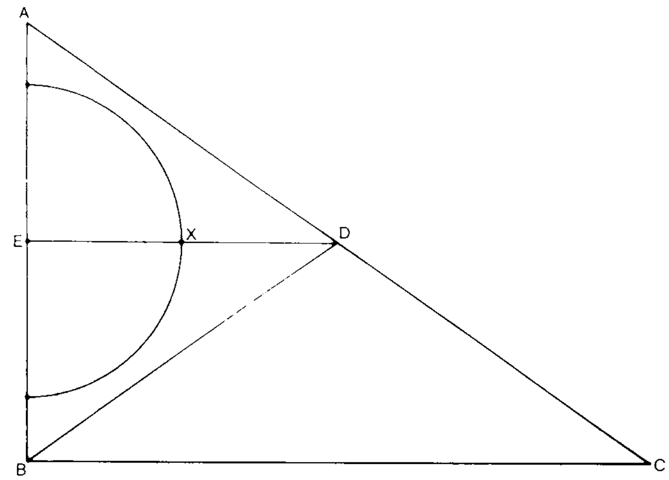
2. with a straight line, connect the vertex b with the median point of the hypotenuse d,



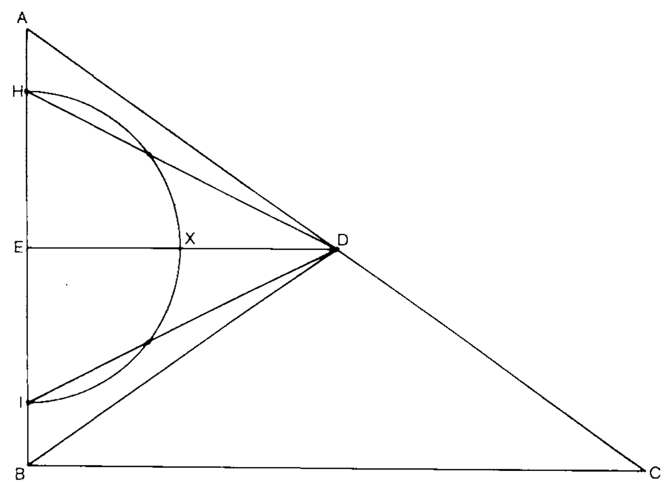
3. trace the straight line d-e (median point of the cathetus a-b),



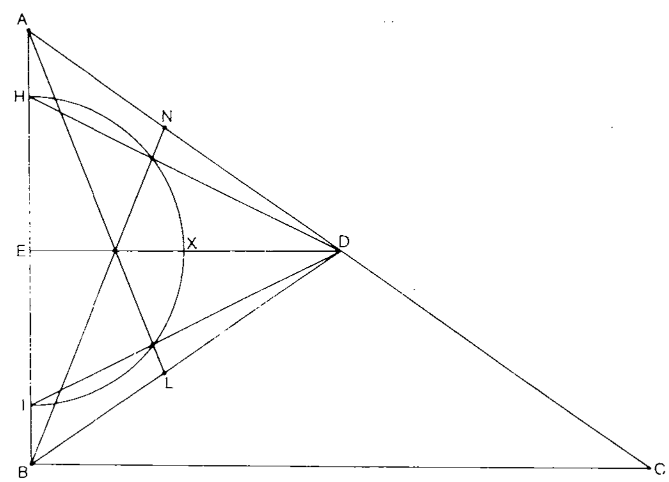
4. describe the semi-circle of ratio e-x (half of e-d),



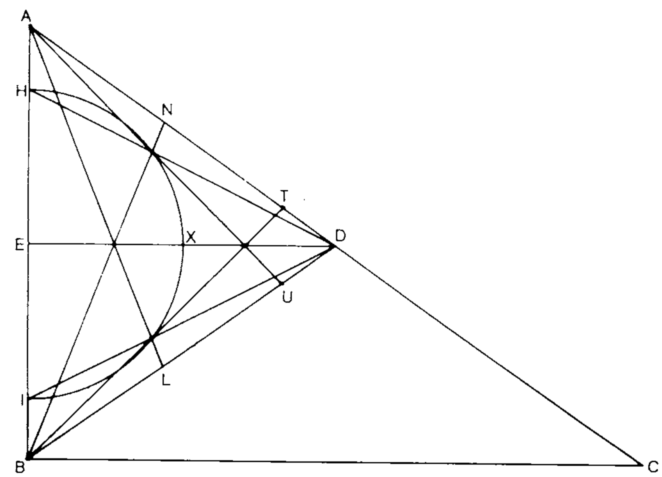
5. trace the straight lines i-d, h-d,



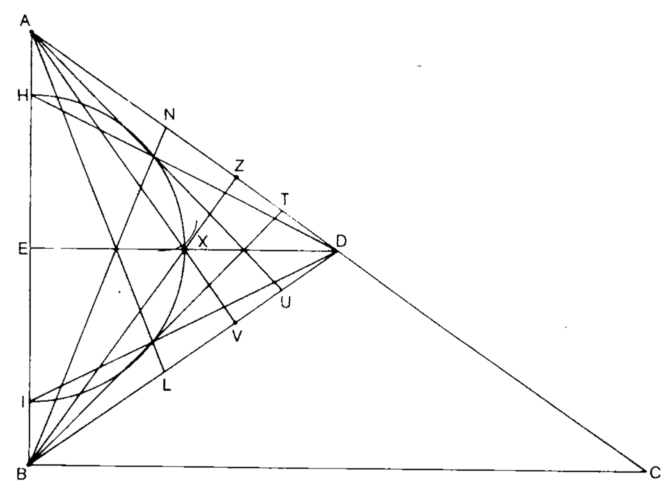
6. a-l, b-n,

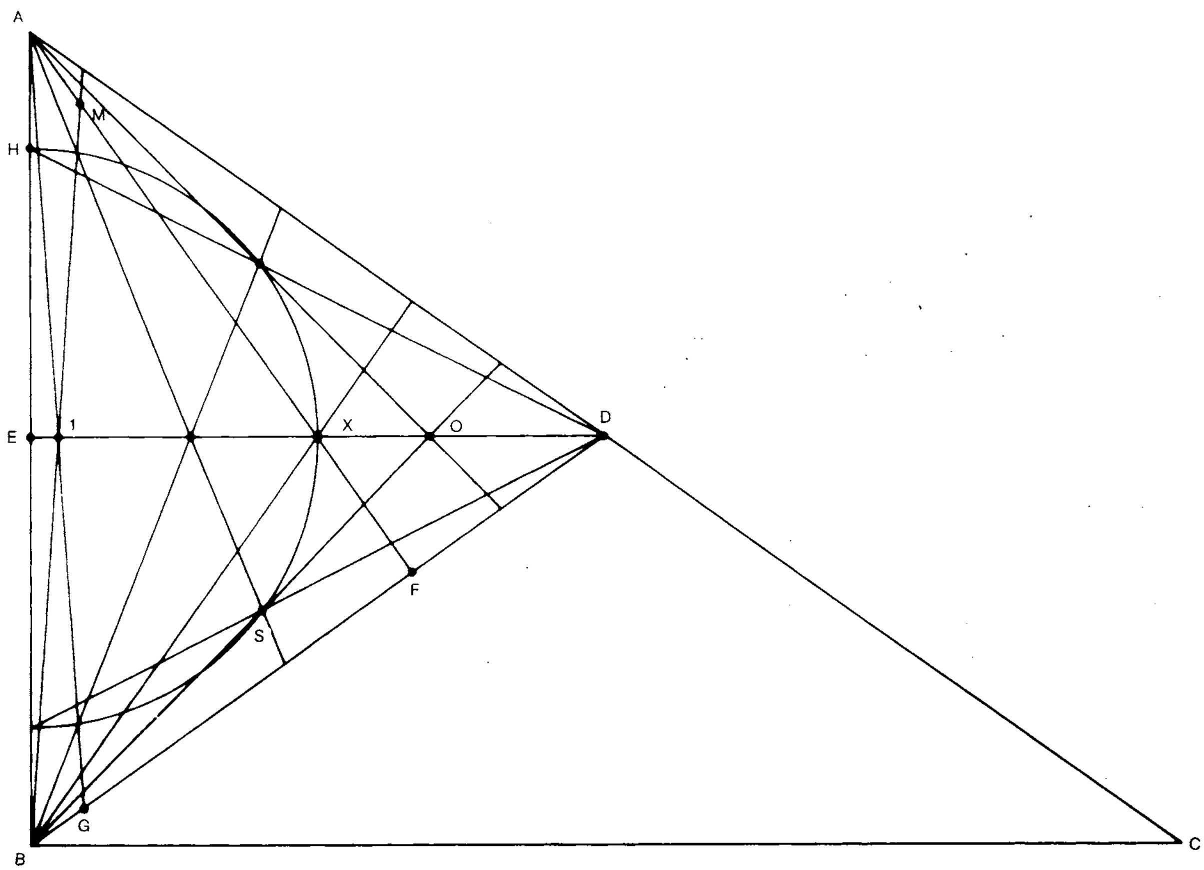


7. b-t, a-u,



8. v-z.

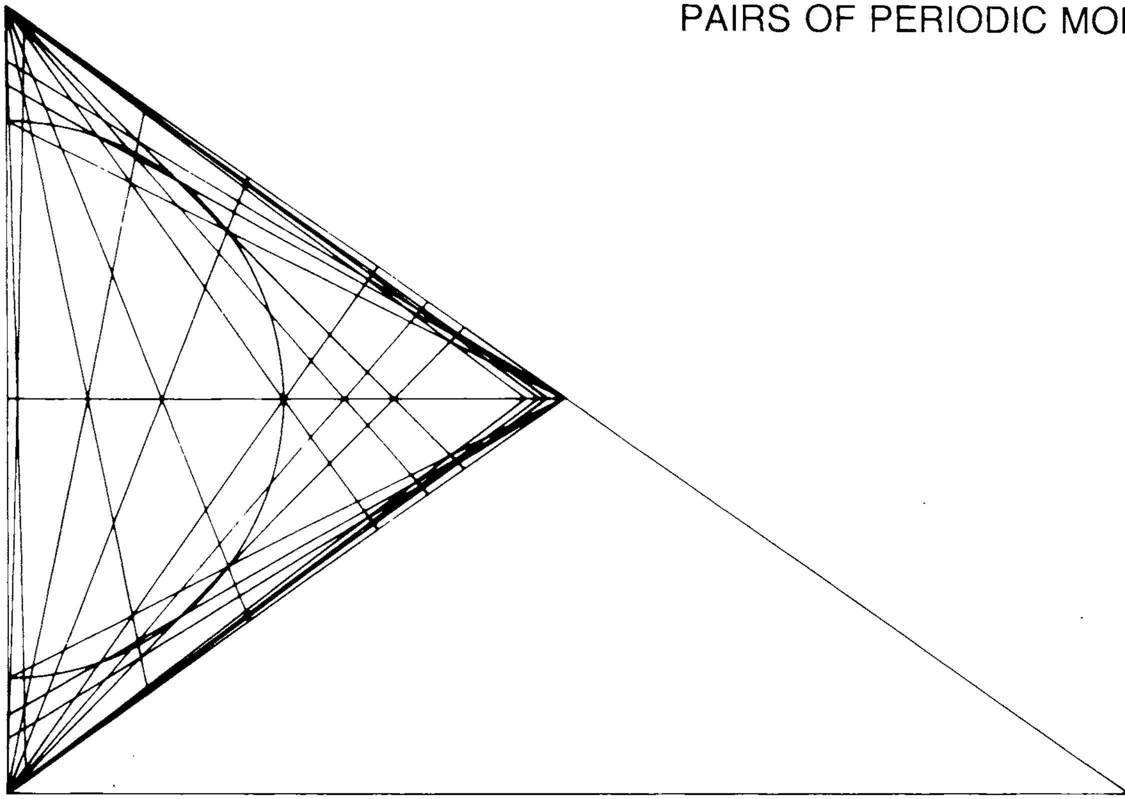




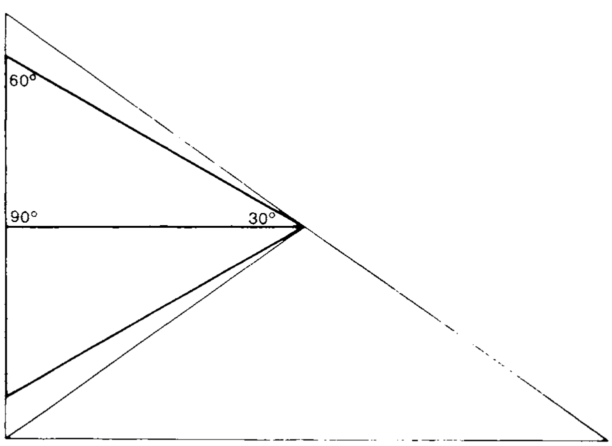
AB = side of the equilateral triangle.
AD = height-median of the equilateral triangle.
ED = heigth between two edges of the tetrahedron.
AF = height of the tetrahedron (vertex-center of the face).
AB = height, side, edge of the cube.
AC = diagonal, height between two vertexes of the cube.
AO + AO = diagonal of the square.
AB = side, edge of the octahedron.
AO + AO = height between two vertexes of the octahedron.
EO + EO = height between two edges of the octahedron.
AB = side of the dodecahedron.
AX + AX + MX + MX = height between two faces of the dodecahedron (between the centers of two faces).
AG + AG = height between the centers of two faces of the dodecahedron with side H – B.
ED + ED + ID + ID = height between two vertexes of the dodecahedron.

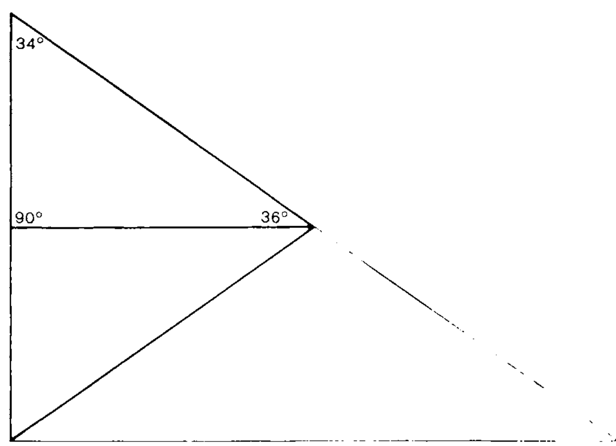
AB = side, edge of the icosahedron.
AS + AS = height between the centers of two opposite faces of the icosahedron.
AG + AG = height between two vertexes of the icosahedron.
HB = ratio (vertex-center) of the regular pentagon (face of the dodecahedron).

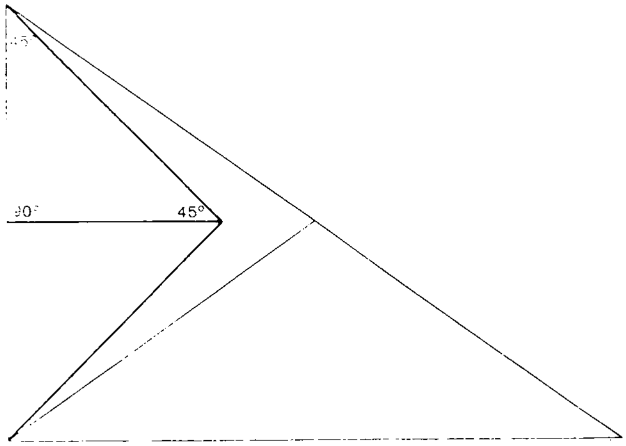
PAIRS OF PERIODIC MODULES

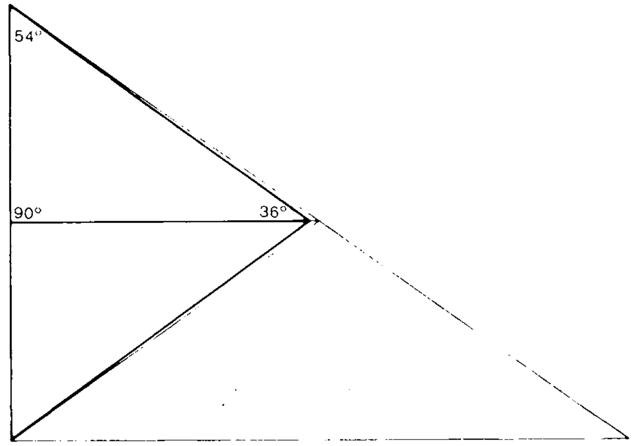


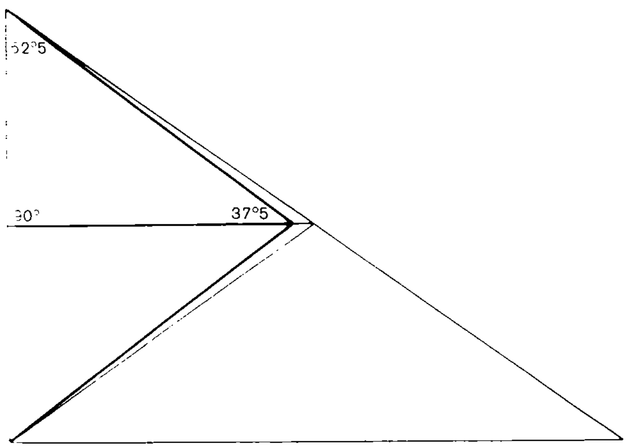
Rectangular triangles which constitute the 40 faces of ten tetrahedral modules.

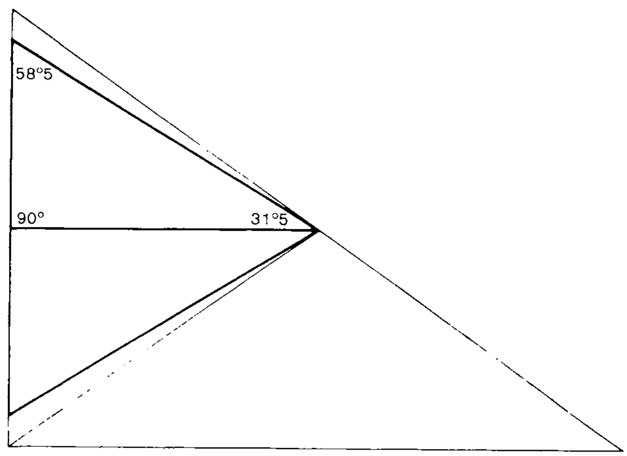


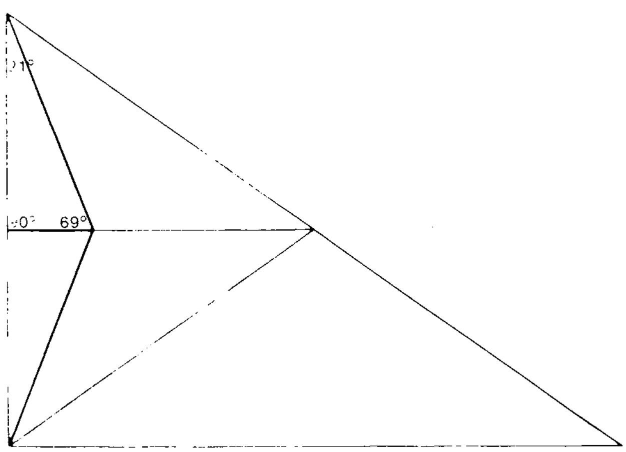


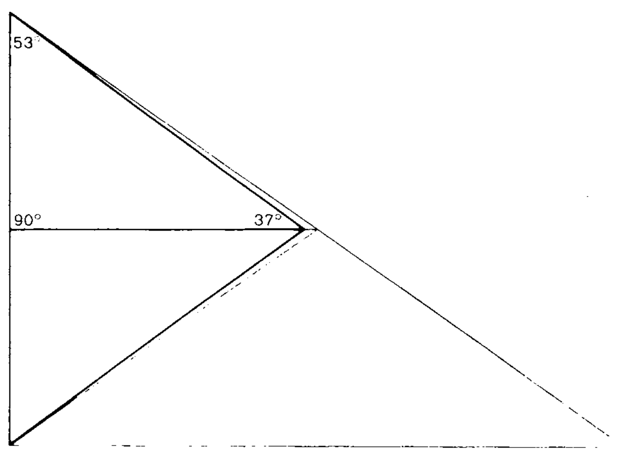

45°


54°
90°
36°

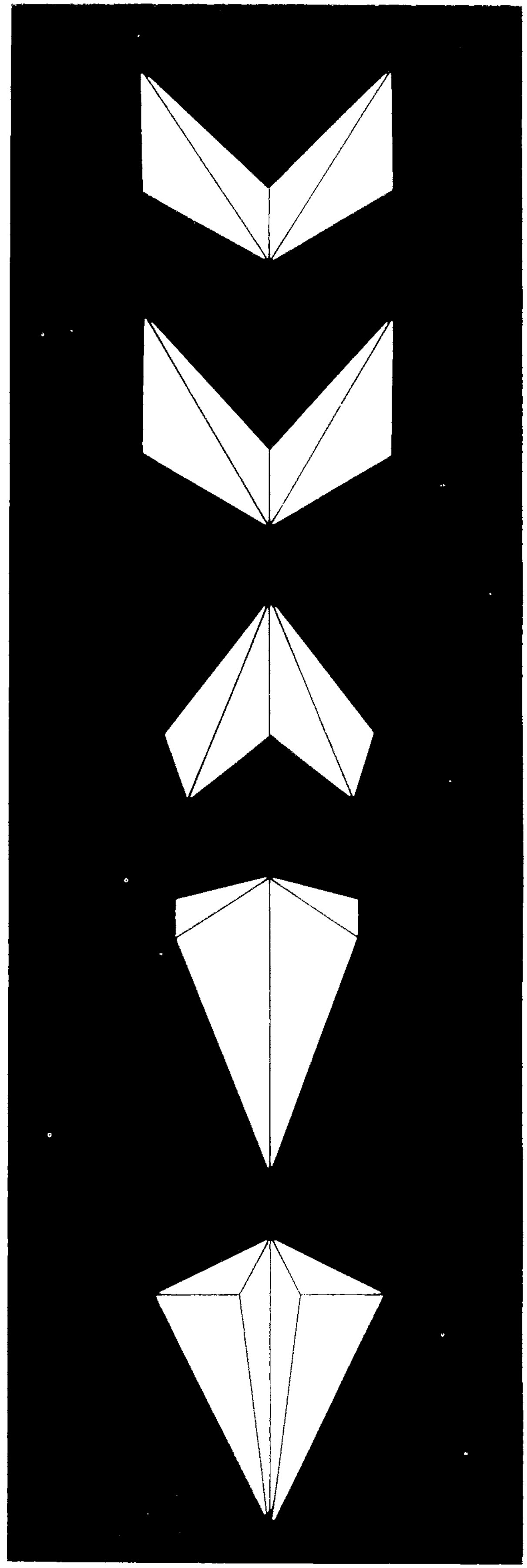

37°5

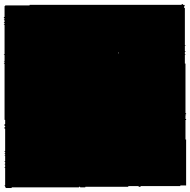

58°5
90°
31°5

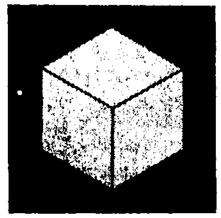

69°

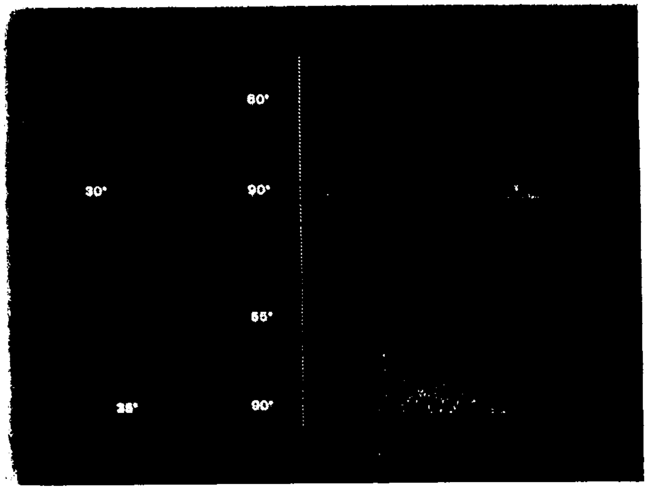

53°
90°
37°

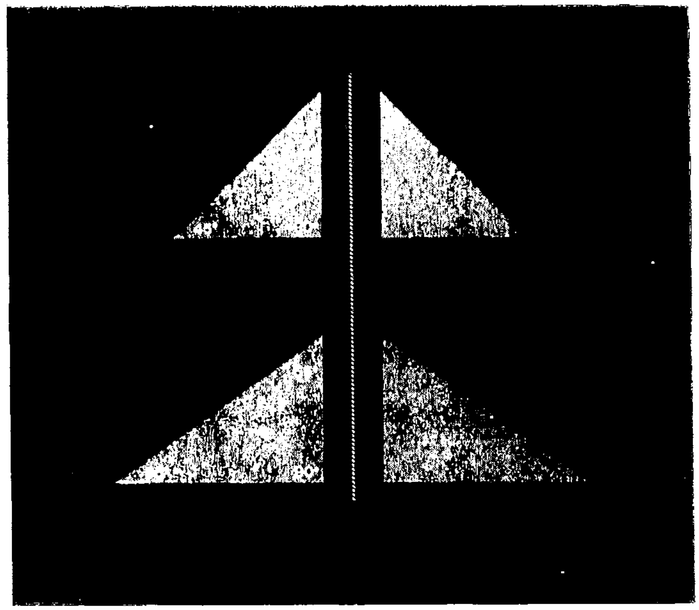
Five pairs of tetrahedral modules in which each unit is the enantiomorphous or antipode of the other. Joined together they form folding chains that can be exactly contained in the five polyhedral cells.





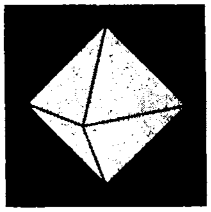


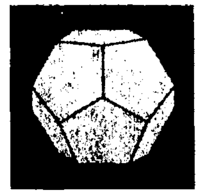


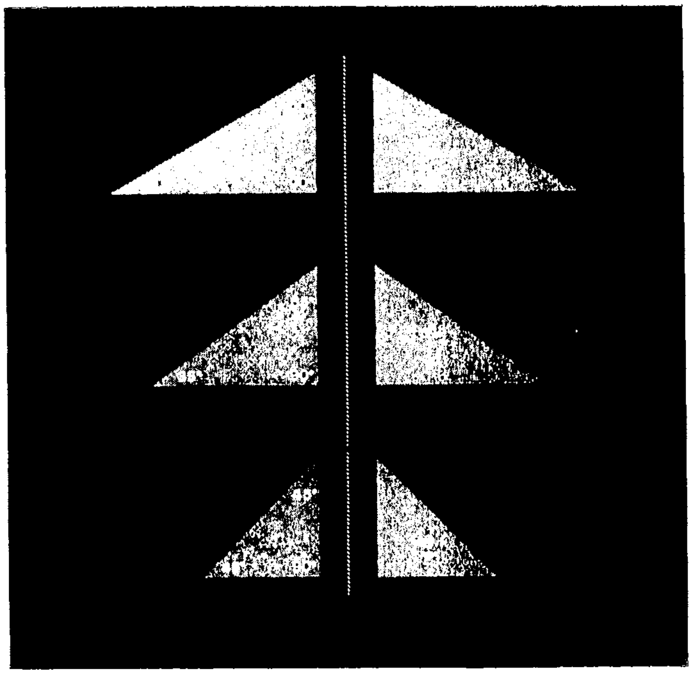


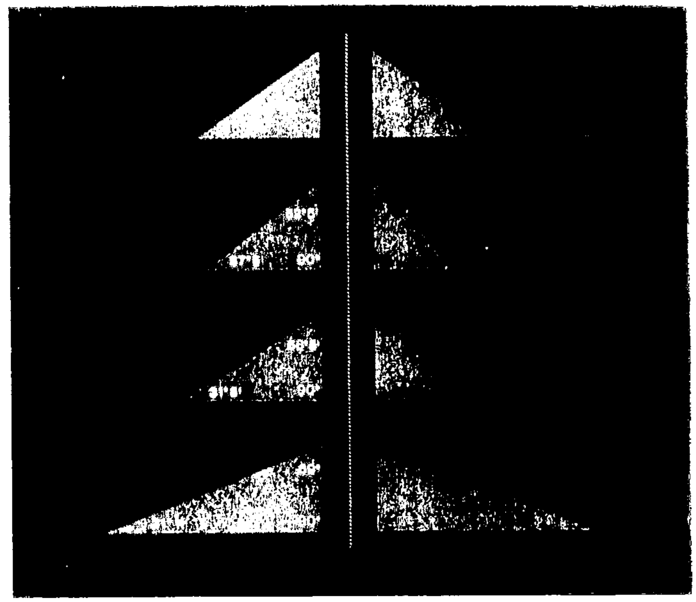
Tetrahedron
A module is constituted by 4 rectangular triangles (2A + 2b); a pair, by 8 specular rectangular triangles (two tetrahedrons); 4 specular modules are contained in the tetrahedral cell; 8 rectangular triangles *A* configurate the external surface of the tetrahedral cell; 8 rectangular triangles *b* form the internal surface.

Cube
A module is constituted by 4 rectangular triangles (2B + 2b);
a pair, by 8 specular rectangular triangles (2 tetrahedrons);
6 specular modules are contained in the cubic cell;
12 rectangular triangles *B* configurate the external surface of the cell;
12 rectangular triangles *b* form the internal surface.



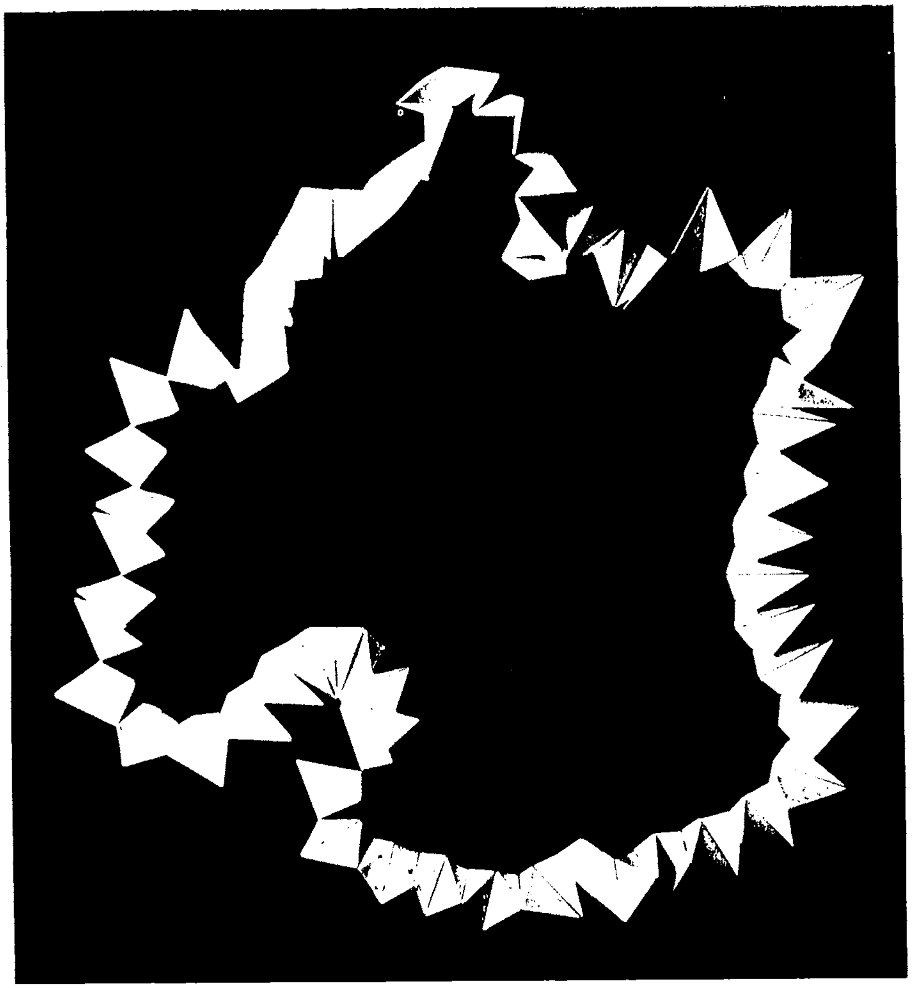


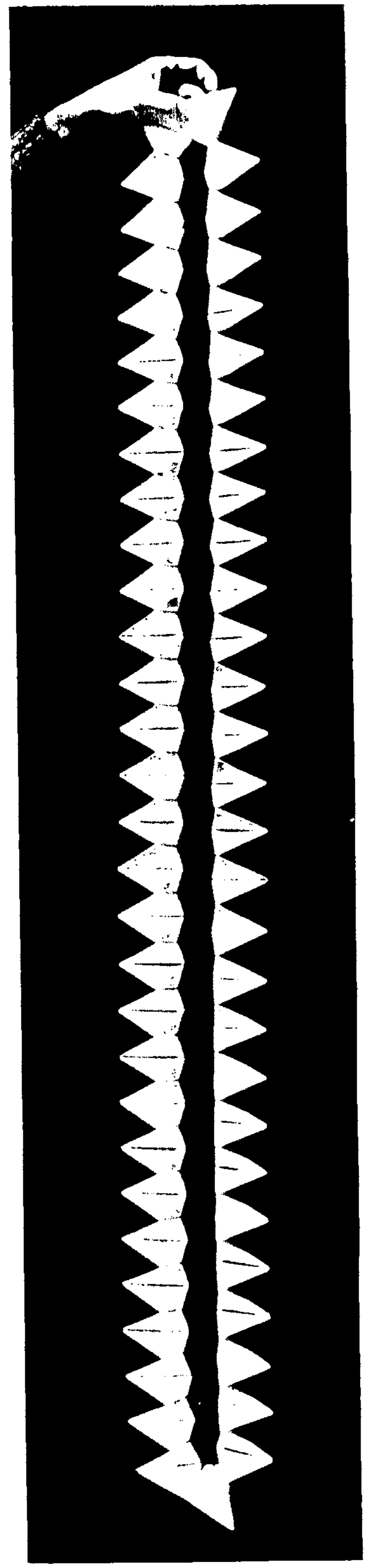




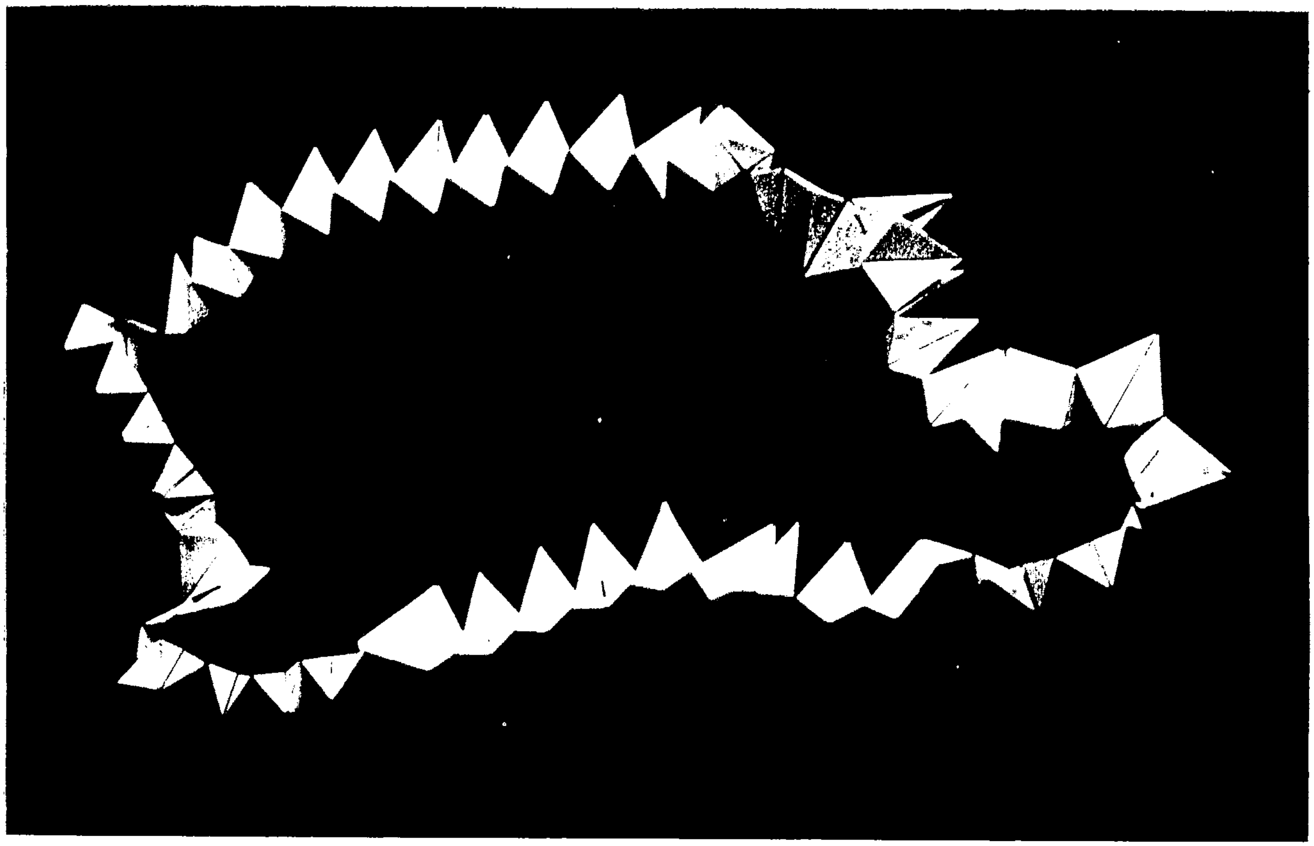
Octahedron
A module is constituted by 4 rectangular triangles (1A + 1b + 2B);
A pair of modules is constituted by 8 specular rectangular triangles (2 tetrahedrons);
16 specular modules are contained in the octahedral cell;
16 rectangular triangles *A* constitute the external surface of the cell;
32 rectangular triangles *B* of which 16 of *B* plus 16 triangles *b* , form the internal surface.

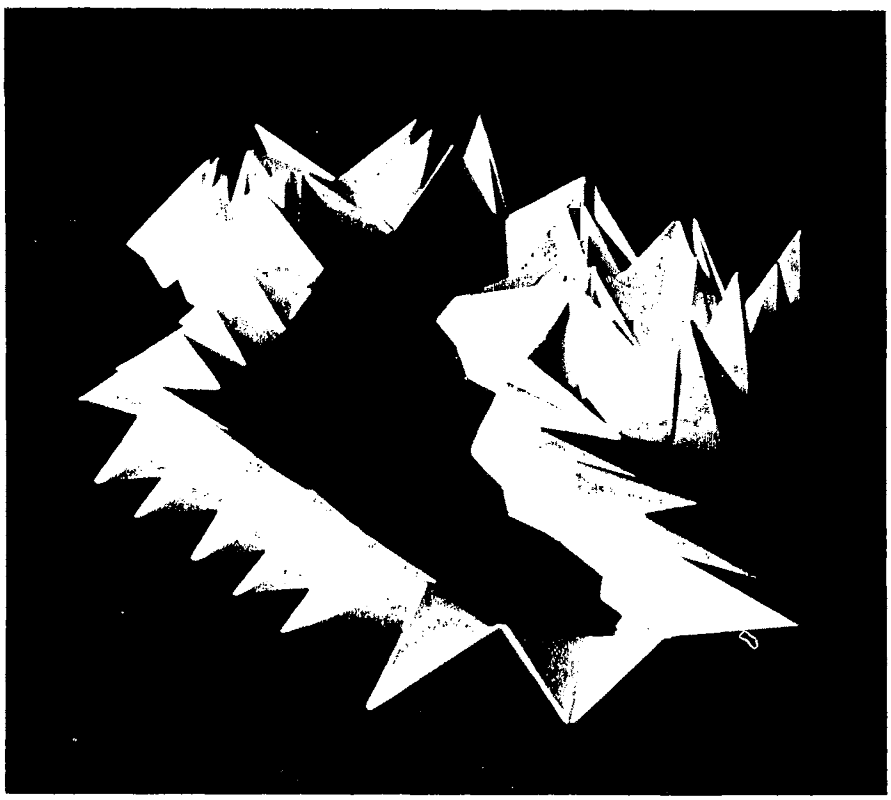
Dodecahedron
A module is constituted by 4 rectangular triangles (1C + 1C + 1C2 + 1C3);
a pair is constituted by 8 specular rectangular triangles (2 tetrahedrons); 120 rectangular triangles C form the external surface of the cell;
360 rectangular triangles *c1, c2, c3,* form the internal surface.

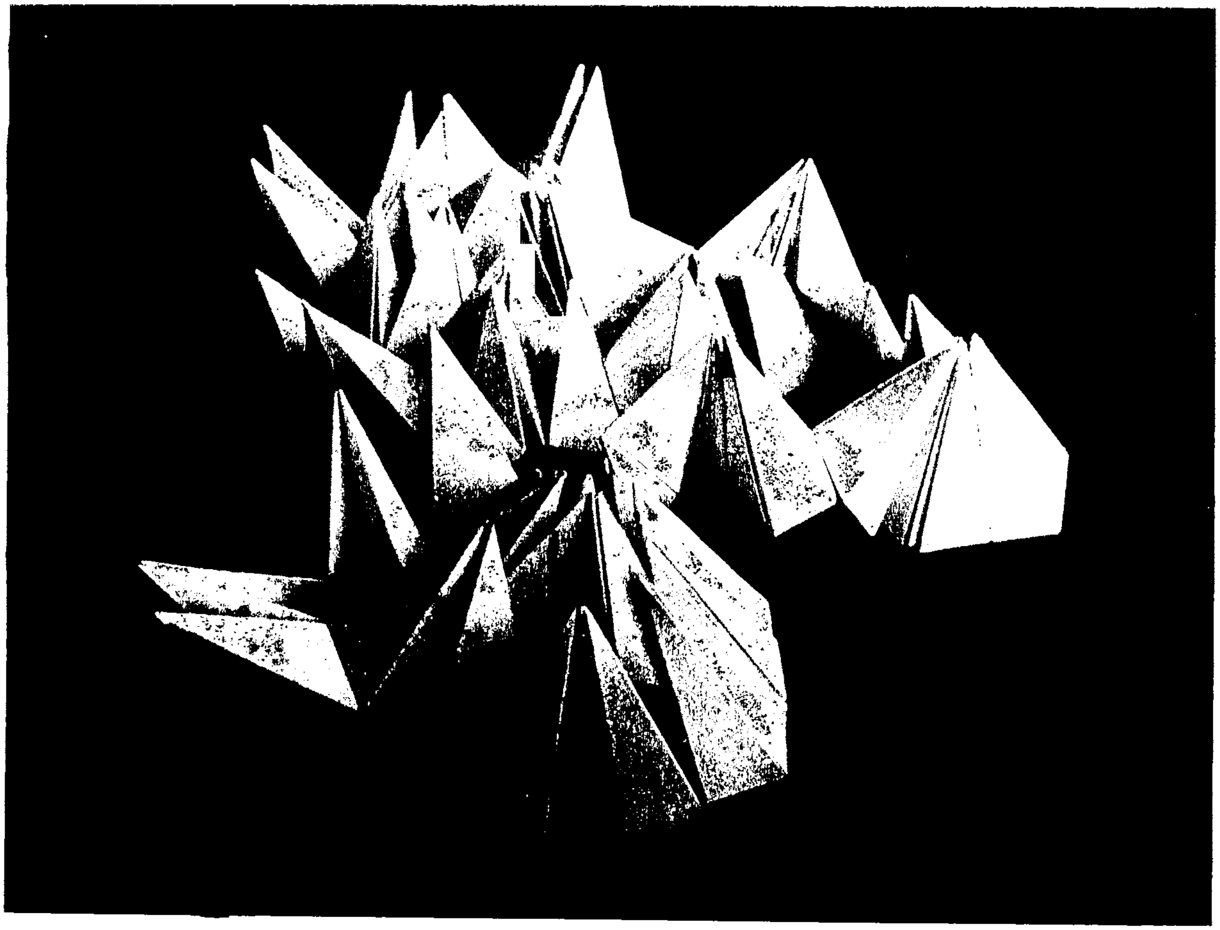


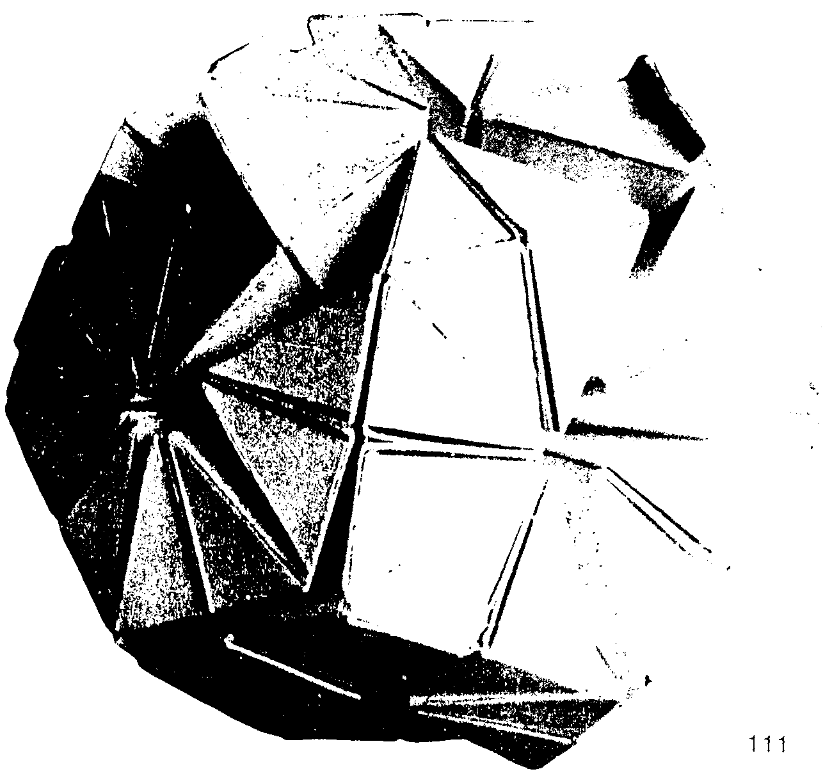


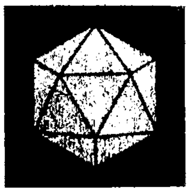
Dodecahedral chain constituted by 120 specular modules: the edge of the dodecahedral cell that contains this model measures 5 centimeters, the modules form a linear sequence whose lenght is about two and half times the height of an average person.

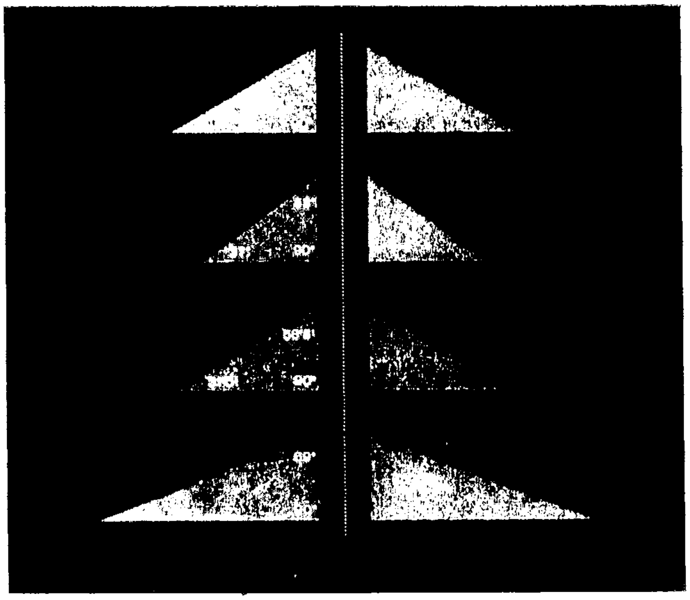






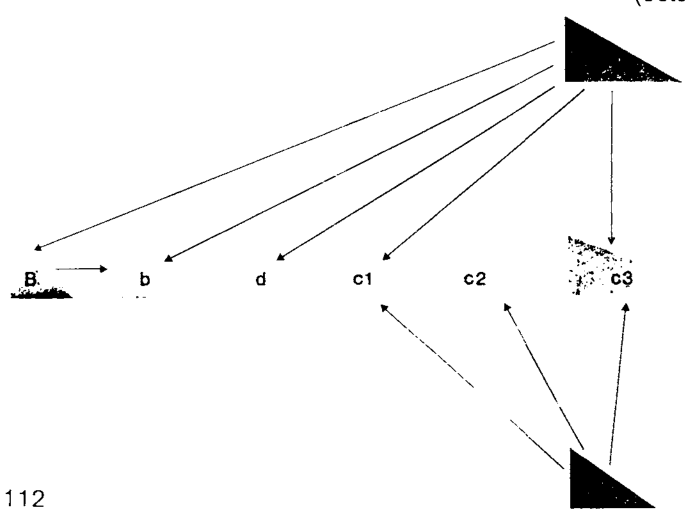


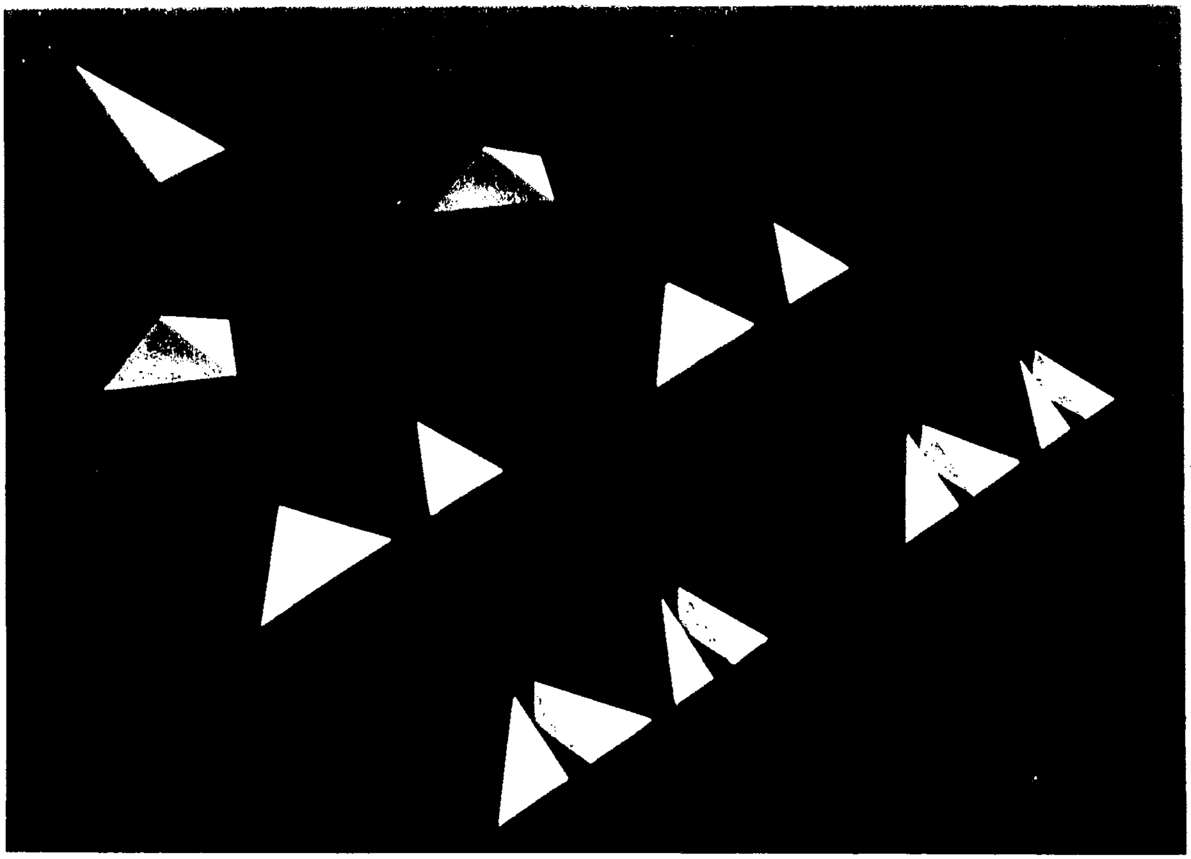




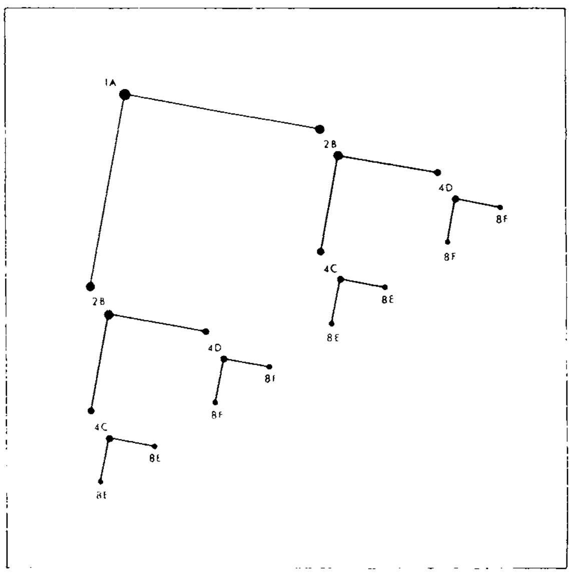
Icosahedron
A module is constituted by 4 rectangular triangles (1A + 1d + 1c + 1c3);
a pair by 8 rectangular triangles *A* which are specular (2 tetrahedrons);
120 triangles *A* form the external surface of the cell;
360 rectangular triangles *d, c1, c2, c3,* form the internal surface.

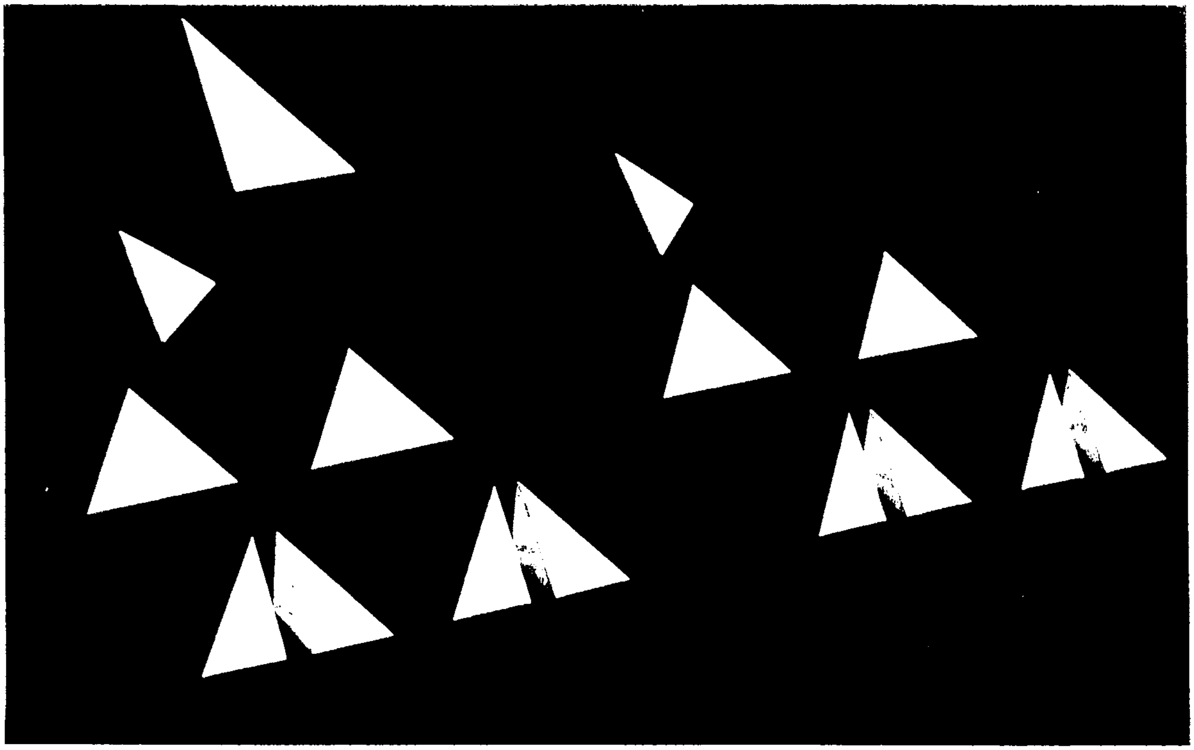
The rectangular triangles constituting the specular modules can be grouped in a scheme which distinguishes the types according to dimensional and angular values, and puts the external in relation to the internal; this has to do with the disposition of the modules in the cells and therefore with the internal and external constitution of the cells themselves. On the vertexes of this triangular diagram we find the rectangular triangles that constitute the total external surface of the five cells.
The rectangular triangle *A* is external face of modules constituting three cells: the tetrahedral, the octahedral, the icosahedral. The rectangular triangle *B* is the external face of the modules that are contained in the cubic cell.
The rectangular triangle *C* is the external face of the modules contained in the dodecahedral cell.
The rectangular triangles that form the internal surface of the various modules and therefore of the cells present themselves in the following relationship with the 3 rectangular triangles of external surface: B, b, d, C2, C3, link with *A* in the relationship of 1:5; C1, C2, C3, link with *C* : relationship 1:3; b, links to *B* : relationship: 1:1.
B is the only rectangular triangle which is present both in the outside (cube) and in the inside (octahedron) of a cell.



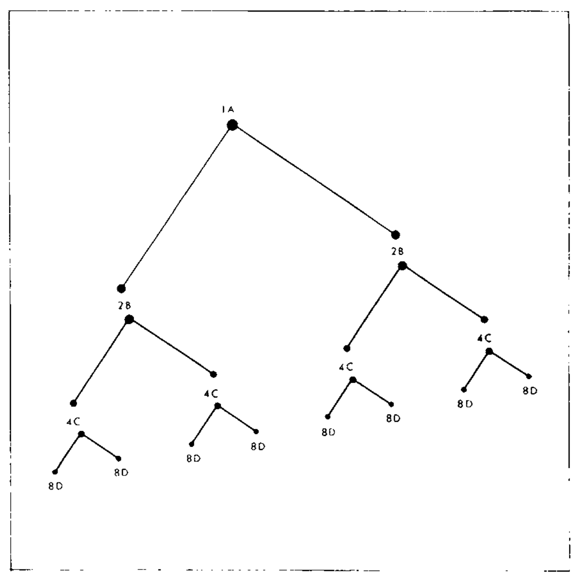


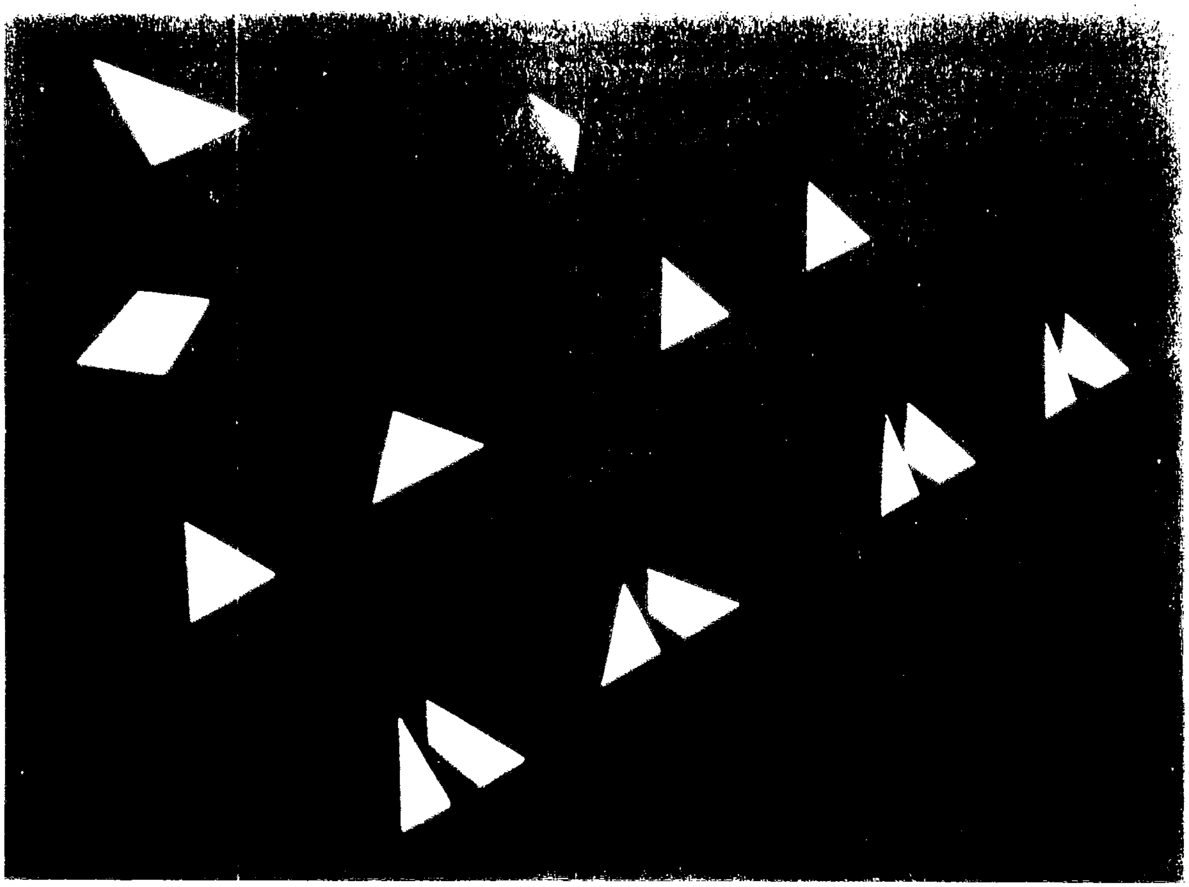
By sectioning in two, four, eight pieces, one component of each of the five pairs, does not matter which of the two, we obtain 20 pairs, of which 13 are analogous to the initial pair. In other words, three sections, following a determined branch of the diagram, originate at least two modules having enantiamorphosous specularity, that is, they have the form of the two modules that compose the initial pair and are one eighth of the volume of the sectioned module. In all five cases, to produce sub-modular pairs out of the basic pair, starting from one of the two components, we must get to the fourth sector of the branched diagram in which a quartet of pairs or an octet of modules is being formed. We have therefore a regularity which is repeated every eight modules, or four pairs of modules. This periodic recurrence constitutes a fugue towards an ever-reducing dimensionality, besides the fact that it establishes a relation between modules, which, in a scale of the polyhedral volumes, can assume their own very precise position and function.

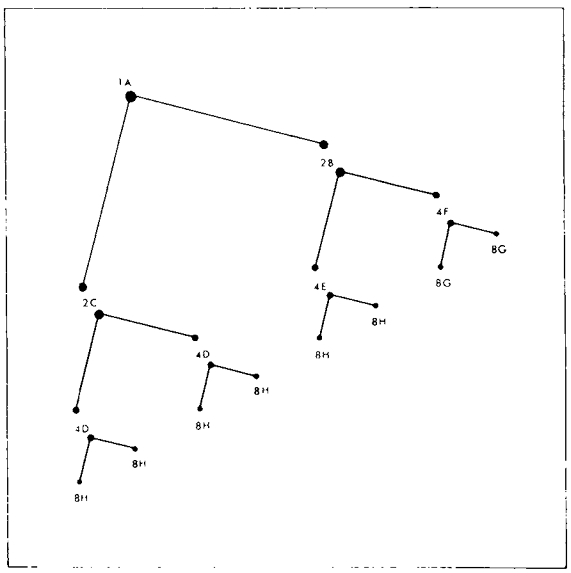




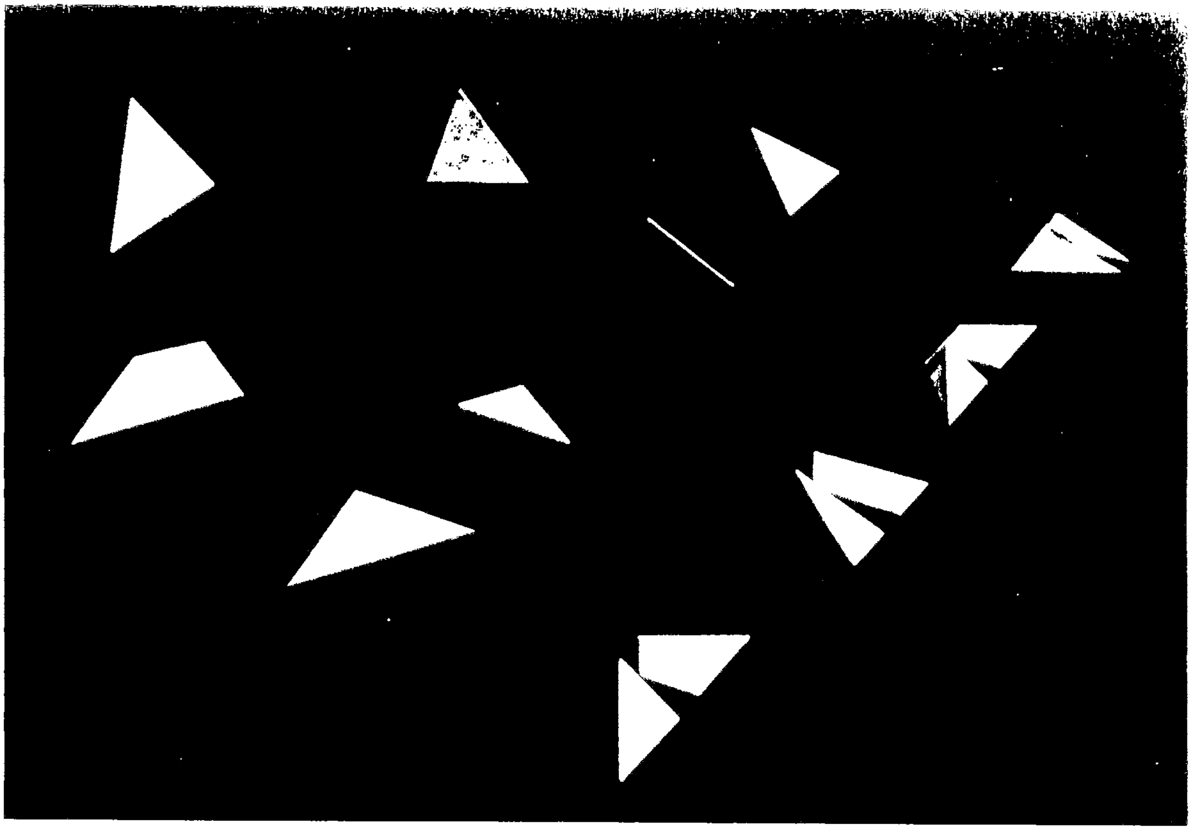
Cube
The four pairs (8D) are symmetrical with respect to each other, analog to the basic pair.



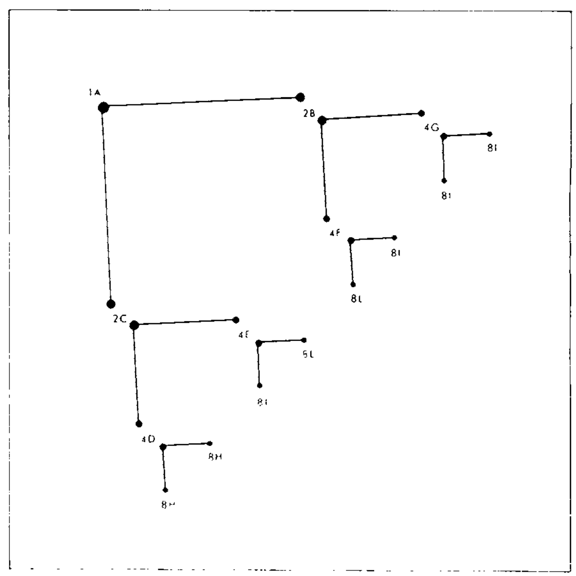


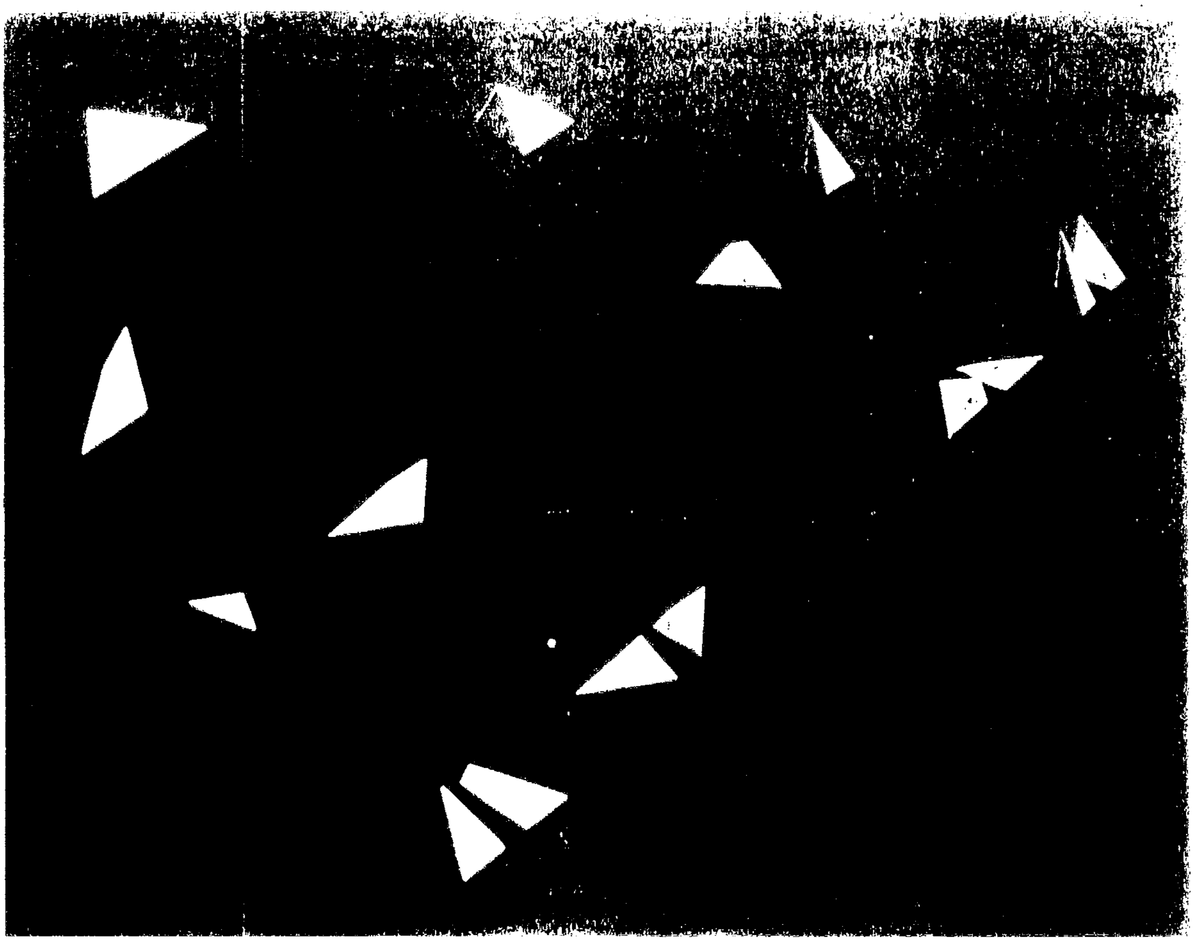


Octahedron
Three pairs (8H) are submodular, with the fourth pair (8G) we can construct a chain precisely foldable in the octahedral cell.

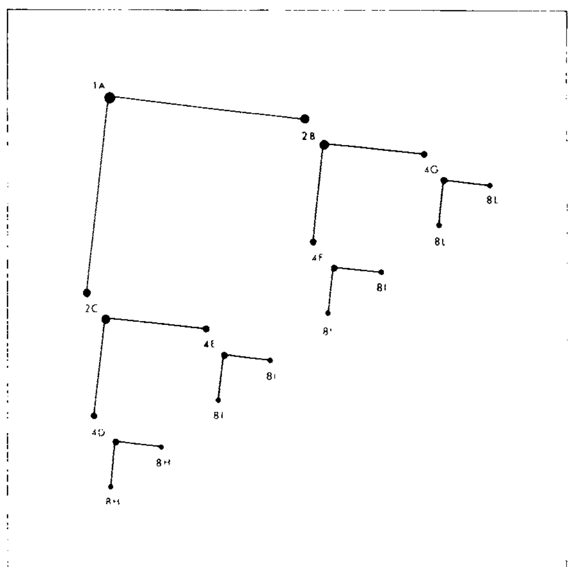


Dodecahedron
Two pairs (8I) are submodular, the pairs 8L, 8H can occupy hyperspaces of interconnected cells.





Icosahedron
Two pairs (8I) are submodular, the pairs 8L, 8H can occupy hyperspaces of interconnected cells.



DEFINITION OF THE OPERATION

We can assume that the infinite possibilities of modulation of a few elementary basic forms, on the plane and in space, be implicitly related to some ordinatory principle, regarding their transformation, development and organization.
The purpose of this work is the research of fundamental ordinatory components, through which be possible to regulate, orient, in a logical way and in a continuity of connections, the tranformation, the development, the modular organization.
The problem must be approached by starting from the principal components which are:
A. a base geometrically given, possibly the most elementary one;
B. the programming of its development; that is how to determine its transformation and its plane, spatial, and temporal growth.
A more detailed list of single problems and of the more relevant operations includes:
1. sectionings of the basic geometric unit;
2. choice of the module or of a pair of modules produced by the discomposition;
3. symmetry operations on a plane with reticular structure;
4. cycles of development on the plane, as a function of the three-dimensional development;
5. three-dimensional development;
6. quantity and modular differenciation:
a. how many modules need to be constructed, of what type, and why;
b. in what way they have to be utilized and for what purpose;
7. spatial orderings;
These points examplify a way to constitute a work plan, articulating a series of successive phases, starting from the basic unit and arriving to various degrees of differenciation, structuralization and modular variability.
It must also be remembered that:
1. the simplest discomposition of the basic module is its section in two parts; each one of the two parts can again be bisected; and so on;
2. each module can be composed of smaller modules;
3. the role of component can be exchanged with that of composed and that of composed into component: in this closed chain a minimum and a maximum of discompositions are present;
4. discomposition and recomposition are alternable, as a function of the typology and modular quantity;
5. the properties of symmetry constitute an essencial instrument in trying to schematically unify the numerous relations which are present between the basic elementary units, the fold-out configurations on the plane and in three-dimensional space, and the systems of articulated modules.
It will also have to be taken into account, as far as possible, the innumerable factors which in nature are constitutive of the development of forms and organisms; the gathered data (scientific information, direct observations) will be utilized in an analogic sense in the constructions of models. We will try to proceed with experimental intent in the formation of the work plan which will have to present itself as an instrument objectively usable, which offers operational variables and which can be modified in relation to new emerging problems.

Illustration sources

Deutscher Taschenbuch Verlag, Munich: p. 10 (b).
A. Feininger: p. 9 (b).
G. Cireddu: p.12 (b).
A. Kleinschmiedt, in the *111th Congress of Surface Activity,* Colon, 1970: p. 11.
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p. 8 (a): from "Le Scienze", february 1976 (microphotograph by Sugie Higashi-Fujime and Tatsuo Ooi).
p. 8 (b): from "Le Scienze", september 1973.
p. 9 (a): from "Le Scienze", march 1974.
p. 10 (a): from "Le Scienze", october 1974
p. 12 (a): from A. Montù, *Natura e geometria*, ed. Melocchi.
p. 71: from "Le Scienze", july 1973 (microphotograph by Michiko Mitani).
p. 94 (a, b): from "Le Scienze", february 1976.

Collaboration of Giorgio Cireddu for the photographs and Gavino Rassu for the execution of some drawings.

Quaderni di design

La collana, dedicata in particolare a insegnanti e studenti di educazione tecnica, educazione artistica e design, è un prezioso e stimolante strumento di consultazione per chiunque si interessi alla formazione della cultura di oggi.
I singoli volumi, ampiamente illustrati, hanno per argomento i punti nodali della progettazione: la raccolta dei dati, la sperimentazione, l'aspetto fisico e psicologico del progetto, l'informazione culturale e tecnologica relativa a materie e strumenti, la metodologia progettuale, la costruzione di modelli, l'indagine su forme e fenomeni naturali, le regole di coerenza formale, il linguaggio tecnico e la comunicazione visiva.

Nella collana

1 Textures
a cura di Corrado Gavinelli
2 La scoperta del triangolo
a cura di Bruno Munari
3 Ricerca e progettazione di un simbolo
a cura di Pietro Gasperini
4 Xerografie originali
a cura di Bruno Munari
5 Modelli di geometria rotatoria
a cura di Giorgio Scarpa
6 La scoperta del quadrato
a cura di Bruno Munari

In preparazione

7 Colore: codice e norma
a cura di Narciso Silvestrini
8 La scoperta del pentagono
a cura di Aldo Montù

Sezionare una forma basilare, un quadrato o un triangolo, in due o più pezzi, ruotare questi pezzi sul piano fino a combinarli in altro modo: si ottengono nuove forme che, a loro volta si combinano con altre simili.
Da tutte queste combinazioni e rotazioni di elementi di forme nascono ancora altre forme che, a prima vista sembrano molto complesse, in realtà tutto diventa molto semplice, quando si conosce la regola.
Giorgio Scarpa mostra in questo libro tutti i procedimenti logici per ottenere nuove forme a due e a tre dimensioni. La conoscenza di questi processi aiuta la formazione di un pensiero progettuale più attivo e, inoltre, fa capire meglio certi aspetti di forme naturali nate dalla rotazione di elementi modulati.
L'arancia è una sfera la cui forma nasce dalla rotazione dei moduli a forma di spicchio, attorno a un asse rettilineo.

33E.0042 QD 05 Scarpa Modelli geom. rotatoria

ZANICHELLI EDITO[illegible]
Prezzo al pubblico
I.V.A. inclusa
£15.000